# 17. “微美”— 微分方程图之美

牛顿和莱布尼茨创立的微分学是现代科技的最重要奠基石之一。基于 1stOpt 简洁而强大的微分方程数值计算及作图功能，可以得到很多精美而又意想不到的微分方程美图。

应用 1stOpt 的循环常数赋值求解功能（LoopConstant），以 12 组微分方程或方程组为演示实例，以飨读者。

## 17.1 微分方程案例 - 1

该案例是经典的 Lorenz 混沌方程：

$$\begin{cases} \frac{dy_1}{dt} = -10(y_1 - y_2) \\ \frac{dy_2}{dt} = 30y_1 - y_1y_3 - y_2 \\ \frac{dy_3}{dt} = -\frac{8}{3}y_3 + y_1y_2 \end{cases} \tag{17-1}$$

代码 17-1

```
Variable t=[0:0.01:200],y1=0,y2=2,y3=9;
Plot y1[x],y3;
ODEFunction
y1'=-10*(y1-y2);
y2'=30*y1-y1*y3-y2;
y3'=-8/3*y3+y1*y2
```

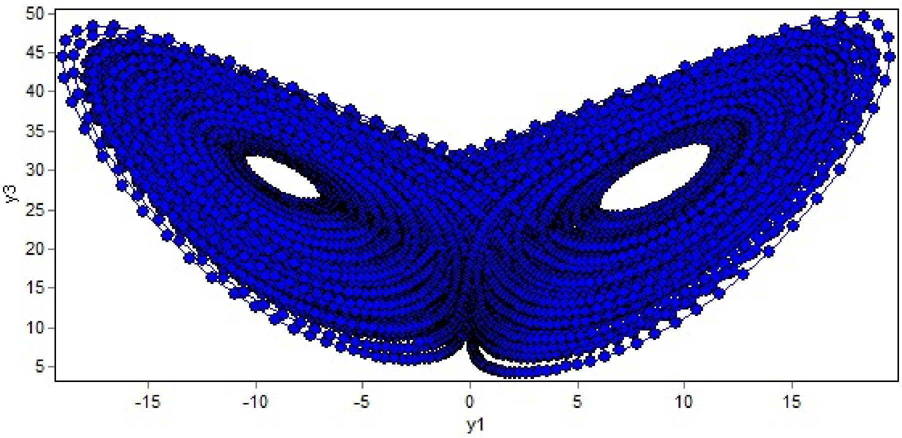



图 17.1 Lorenz 混沌方程图-1

在公式（17-1）的基础上增加一个常数变量 c，其取值分别为[1,2,5,10,15,20]，

通过“LoopConstant”循环计算。

$$\begin{cases} \frac{dy_1}{dt} = -10(y_1 - y_2) \\ \frac{dy_2}{dt} = 30y_1 - y_1y_3 - y_2 \\ \frac{dy_3}{dt} = -\frac{8}{3}y_3 + y_1y_2c \end{cases} \tag{17-2}$$

代码 17-2

```
LoopConstant c=[1,2,5,10,15,20];
Variable t=[0:0.01:200],y1=0,y2=2,y3=9;
Plot y1[x],y3+c;
ODEFunction
y1'=-10*(y1-y2);
y2'=30*y1-y1*y3-y2;
y3'=-8/3*y3+y1*y2*c;
```

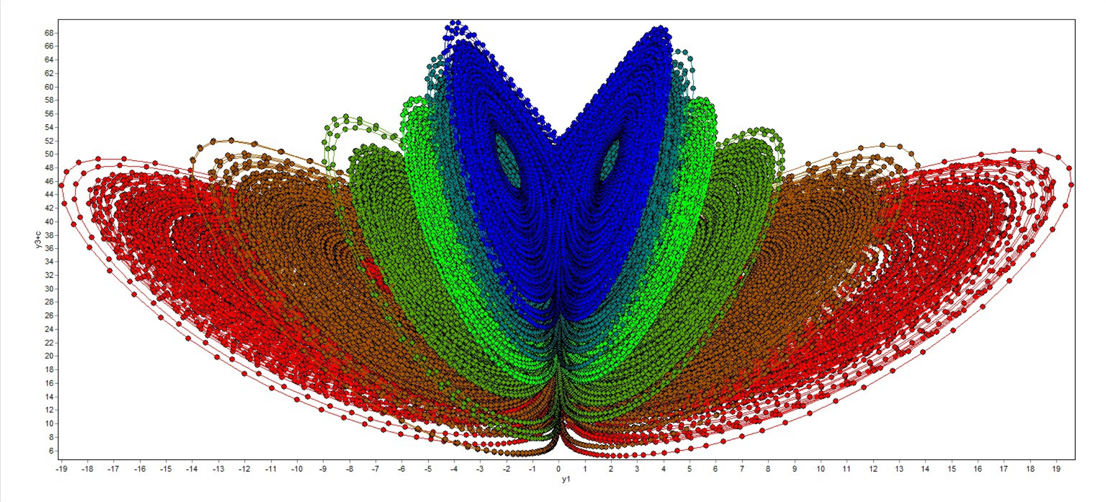


图 17.2 Lorzen 混沌方程图-2

## 17.2 微分方程案例 –2

微分方程：

$$\frac{dx}{dt} = -1.0tx - a\sin(t - ax) \tag{17-3}$$

代码 17-3

```
LoopConstant a=[-1:0.01:2];
Variable t=[0:0.01:2], x=(-a+1)^2;
Plot x[x],x';
ODEFunction x'=-1*t*x-a*sin(t-x*a);
```

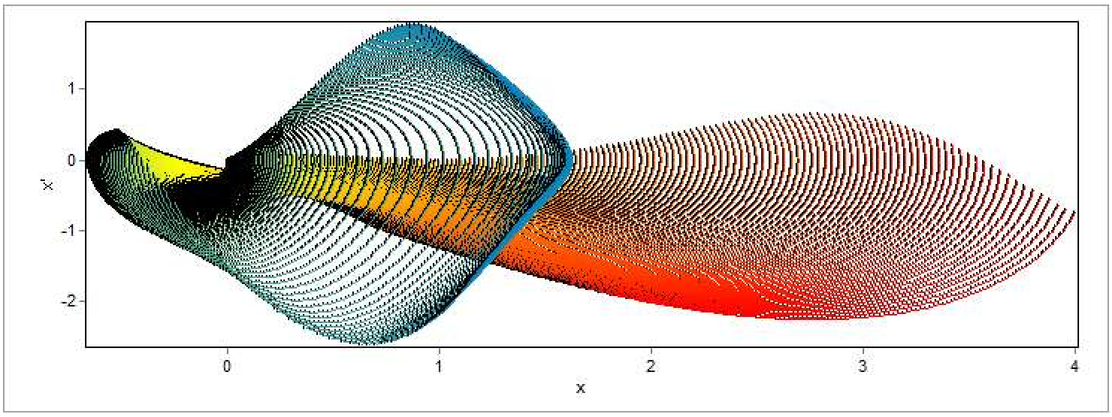



图 17.3  微分方程图-3

## 17.3  微分方程案例  - 3

微分方程组：

$$\begin{cases} \frac{df}{dx} = 3f + 4g \\ \frac{dg}{dx} = -4f + 3g \end{cases} \quad (17\text{-}4)$$

代码 17-4

```
LoopConstant a=[0:0.05:10];
Variable x=[0,3-a/2], f=0, g=0.1-a;
ODEOptions = [SN=100];
Plot f[x],f',g',-f-g,-f'-g';
ODEFunction f'=3*f+4*g;
                    g'=-4*f+3*g;
```

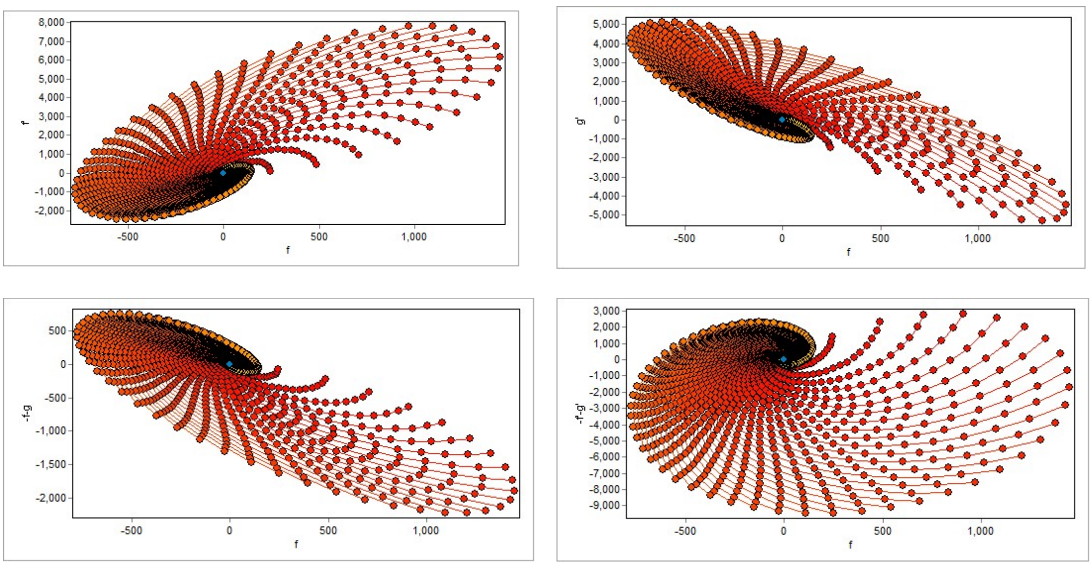



图 17.4 微分方程图-4

## 17.4 微分方程案例 - 4

微分方程组：

$$
\begin{cases}
\dfrac{d^2x}{dt^2} = \dfrac{c\left(-16000t-160-\left(200000x-800\frac{dy}{dt}-80000y+2000\frac{dx}{dt}\right)\right)}{185} \\
\dfrac{d^2y}{dt^2} = \dfrac{c\left(1600t+16-\left(-8000x+80\frac{dy}{dt}-80\frac{dx}{dt}+8000y\right)\right)}{16}
\end{cases}
\tag{17-5}
$$

代码 17-5

```
LoopConstant c=[1:0.05:6];
Variable t=[0:0.01:3.5],x'=-0.012185,x=-0.0021512,y=-0.0021512,y'=-0.2;
Plot x[x],x',c*x'+y'[y2],x'+y',(x'+y*y');
ODEFunction
x''=c*(-16000*t-160-(200000*x-800*y'-80000*y+2000*x'))/185;
y''=c*(1600*t+16-(-8000*x+80*y'-80*x'+8000*y))/16;
```

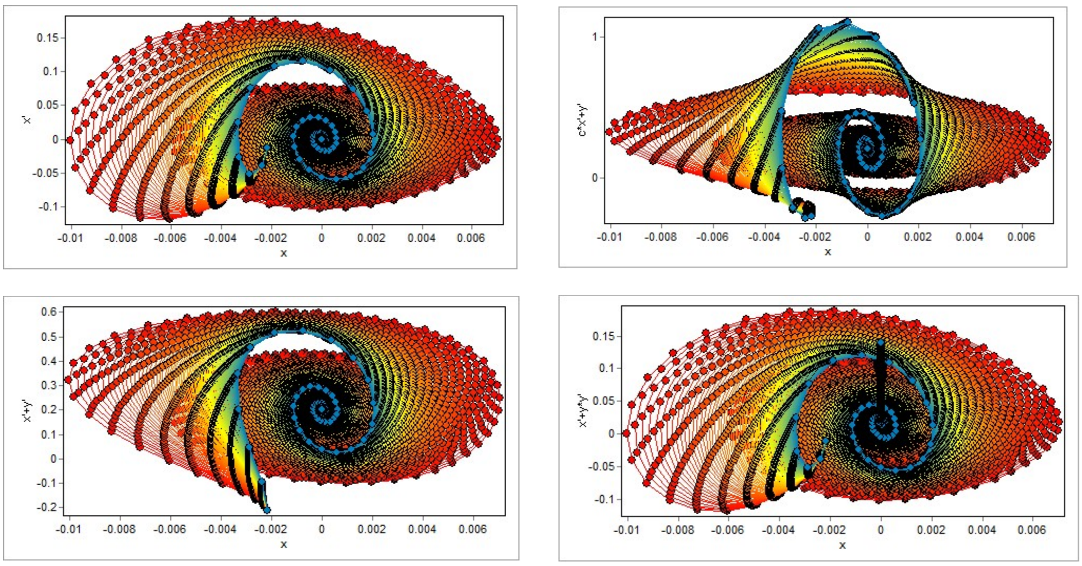



图 17.5 微分方程图-5

## 17.5 微分方程案例 - 5

微分方程组：

$$
\begin{cases}
\dfrac{dx}{dt} = \dfrac{2(\sin(t)-x+y)}{\sqrt{(\sin(t)-x)^2+(\cos(t)-y)^2}} \\
\dfrac{dy}{dt} = \dfrac{2(\cos(t)-y+x)}{\sqrt{(\sin(t)-x)^2+(\cos(t)-y)^2}}
\end{cases}
\tag{17-6}
$$

代码 17-6

```
LoopConstant a=[-3.5:0.1:3.5];
Variable t=[0:0.05:10],x=a*2,y=-a;
Plot x[x],y,sinh(0.45*y),sinh(0.75*y),sinh(y);
ODEFunction x' = 2*(Sin(t) - x+y)/Sqrt((Sin(t) - x)^2 + (Cos(t) - y)^2);
                    y' = 2*(Cos(t) - y+x)/Sqrt((Sin(t) - x)^2 + (Cos(t) - y)^2);
```

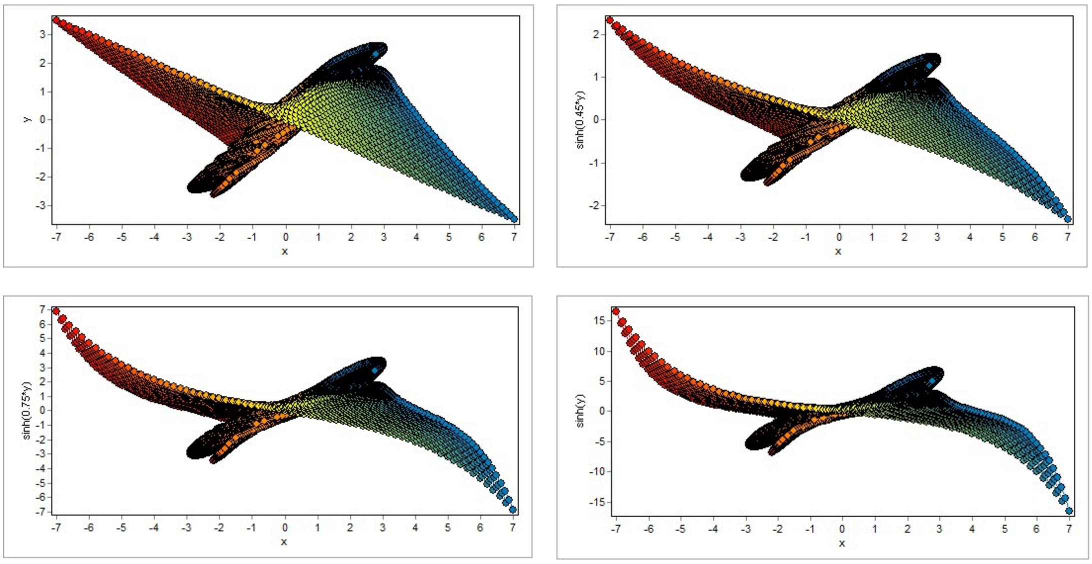



图 17.6 微分方程图-6

微分方程组：

$$\begin{cases} \dfrac{dx}{dt} = \dfrac{2(\cos(t)-x+y)}{\sqrt{(\sin(t)y-x-0.1)^2+(\cos(t)x-y)^2}} \\ \dfrac{dy}{dt} = \dfrac{2(\sin(t)-y+x)}{\sqrt{(\sin(t)y-x-0.1)^2+(\cos(t)x-y)^2}} \end{cases} \tag{17-7}$$

代码 17-7

```
LoopConstant a=[-3.5:0.1:3.5];
Variable t=[0:0.1:10],x=2*a,y=a^2;
Plot x[x],y, y+x/2, y+x, y/2+x;
ODEFunction x'=2*(Cos(t)-x+y)/Sqrt((Sin(t)*y-x-0.1)^2+(Cos(t)*x-y)^2);
                    y'=2*(Sin(t)-y+x)/Sqrt((Sin(t)*y-x-0.1)^2+(Cos(t)*x-y)^2);
```

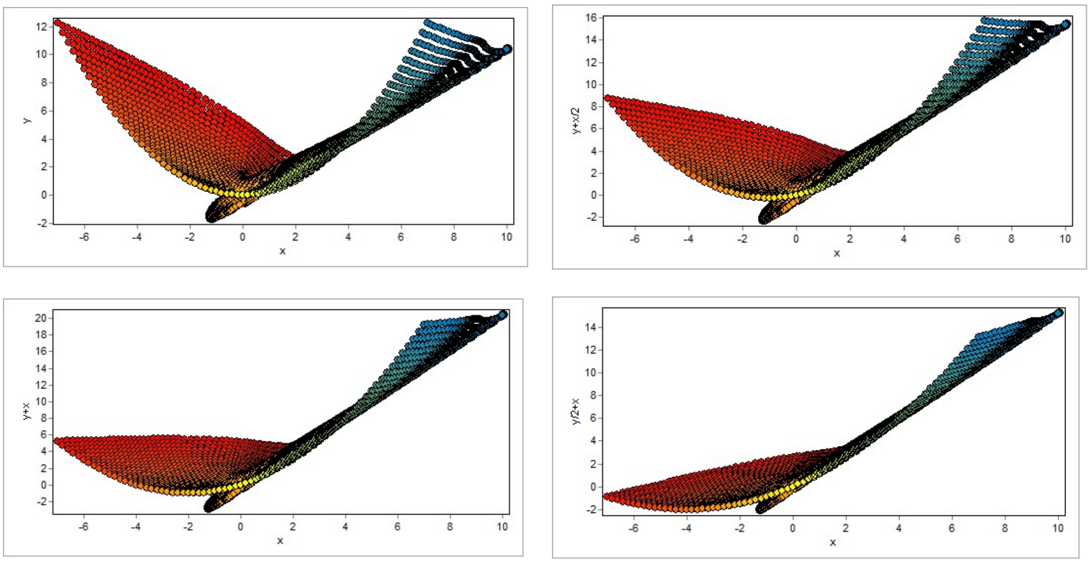



图 17.7 微分方程图-7

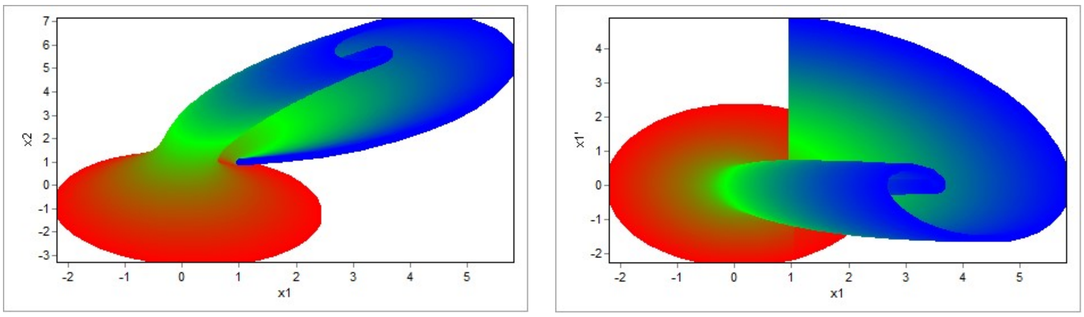
## 17.6 微分方程案例 - 6

微分方程组：

$$\begin{cases} \dfrac{dx_1}{dt} = -0.1x_1 - x_2 + a \\ \dfrac{dx_2}{dt} = x_1 - 0.1x_2a \end{cases} \tag{17-8}$$

代码 17-8

```
LoopConstant a=[-1:0.05:6];
ODEOptions = [SN=200];
Variable t=[0,10*pi],x1=1,x2=1;
Plot x1[x],x2,x1'[y2],0.2*x2-x1';
ODEFunction x1'=-0.1*x1-x2+a;
                    x2'=x1-0.1*x2*a;
```



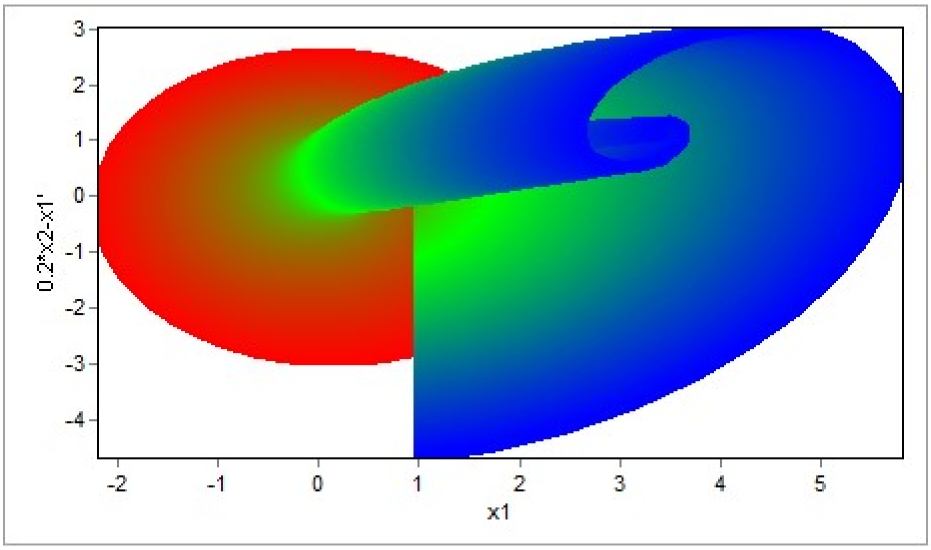



图 17.8 微分方程图-8

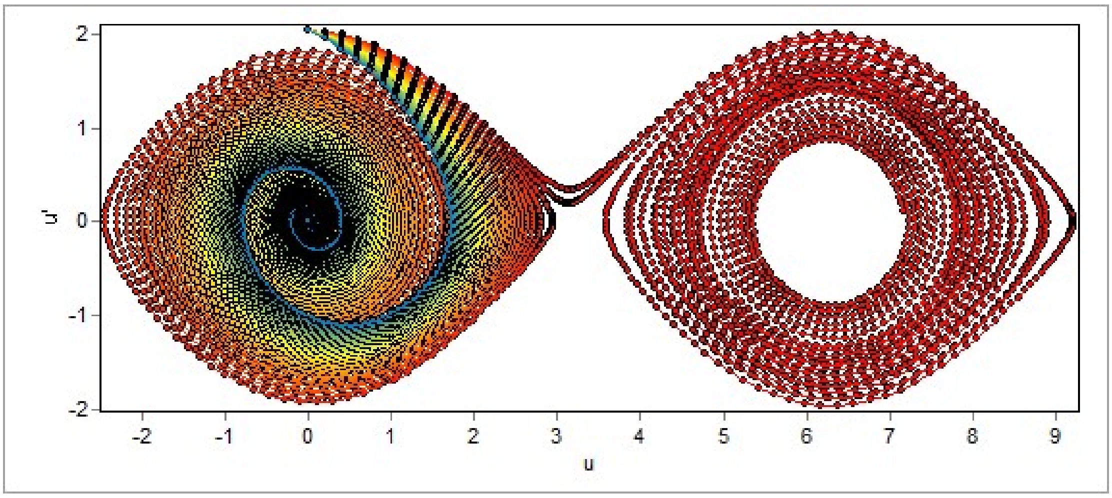
## 17.7 微分方程案例 - 7

微分方程：

$$\frac{d^2u}{dt^2} = -f\frac{du}{dt} - \sin(u) \tag{17-9}$$

代码 17-9

```
LoopConstant f=[0.01:0.01:0.4];
Variable t=[-50,50], u=0, u'=2.05;
ODEOptions = [SN=1000];
Plot u[x],u',0.2*u*u''-0.85*u';
ODEFunction u''=-f*u'-sin(u);
```



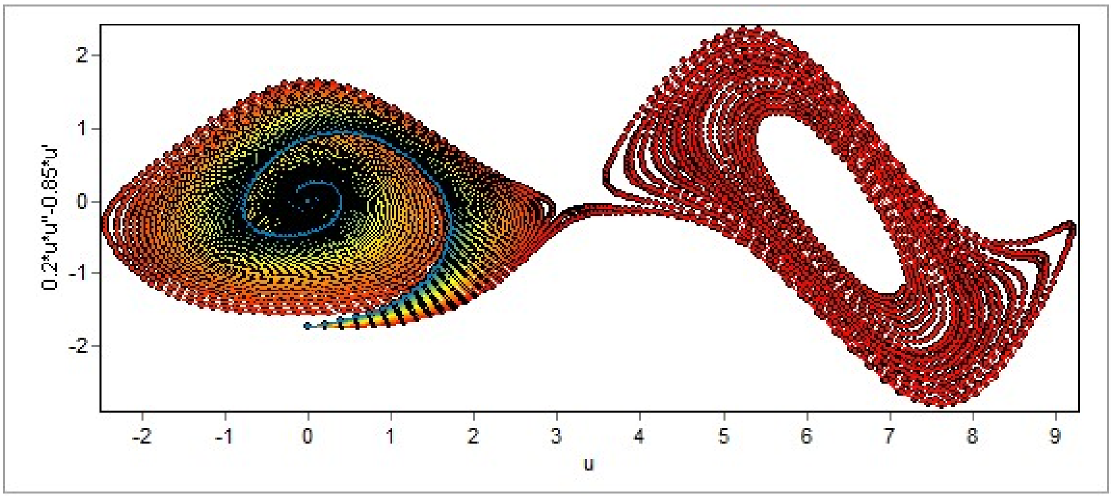


图 17.9 微分方程图-9

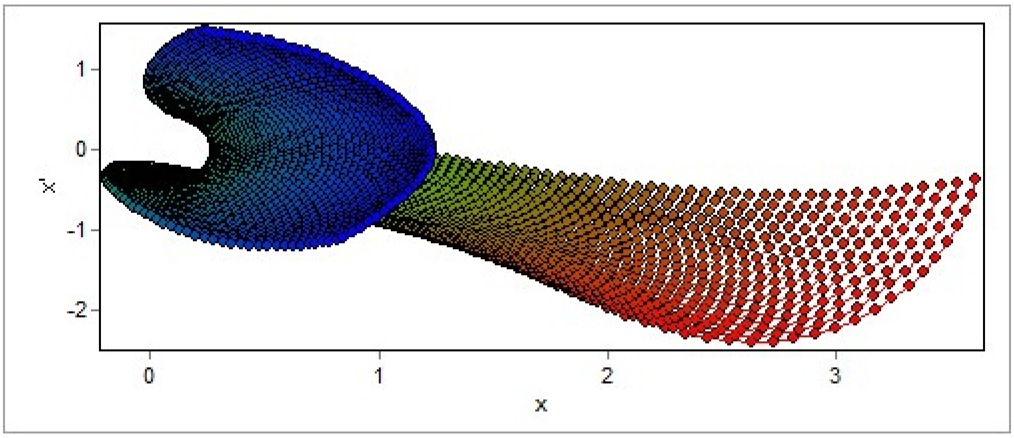
## 17.8 微分方程案例 - 8

微分方程组：

$$\frac{dx}{dt} = -1.0tx - a\sin(t - x - a) \tag{17-10}$$

代码 17-10

```
LoopConstant a=[-0.9:0.02:1.5];
ODEOptions = [SN=50];
Variable t=[0,2], x=(-a+1)^2;
PassParameter c=ln(sin(x')*cos(x)^2+8.5);
Plot x[x],x',c;
ODEFunction x'=-1*t*x-a*sin(t-x-a);
```



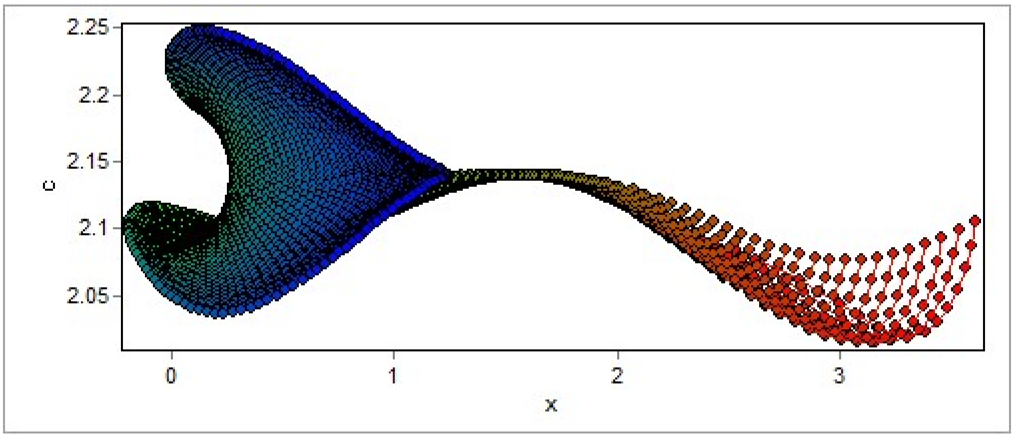



图 17.10 微分方程图-10

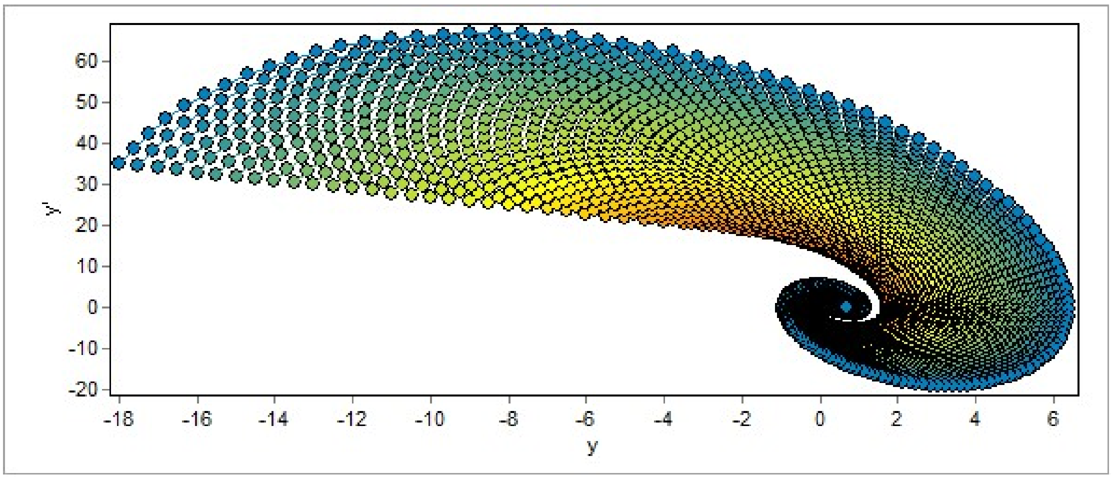
## 17.9 微分方程案例 - 9

微分方程：

$$\frac{d^2y}{dx^2} = -4\frac{dy}{dx} - 29y + a \tag{17-11}$$

代码 17-11

```
LoopConstant a=[0:0.5:20];
Variable x=[0:0.01:5], y=2-a,y'=15+a;
Plot y[x],y',a/100-y';
ODEFunction y''=-4*y'-29*y+a;
```



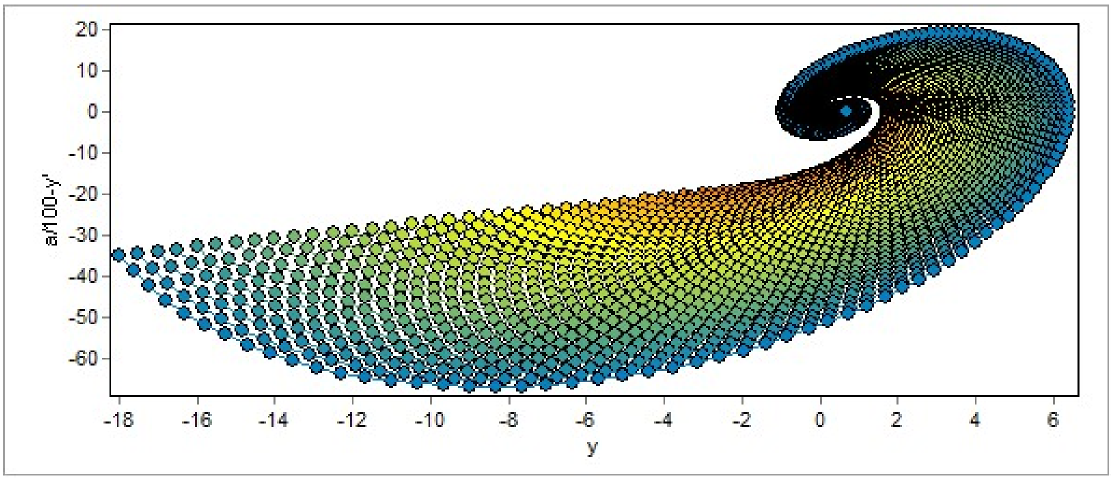



图 17.11 微分方程图-11

## 17.10 微分方程案例 - 10

微分方程：

$$\frac{d^2y}{dx^2} = 2\cos(x) - x\sin(x - a) \tag{17-12}$$

代码 17-12

```
LoopConstant a=[-1:0.02:5];
Variable x=[0:0.1:4], y=0, y'=0;
Plot y[x], y', y'',y*y',y'*y'';
ODEFunction y''=2*cos(x) - x*sin(x-a);
```

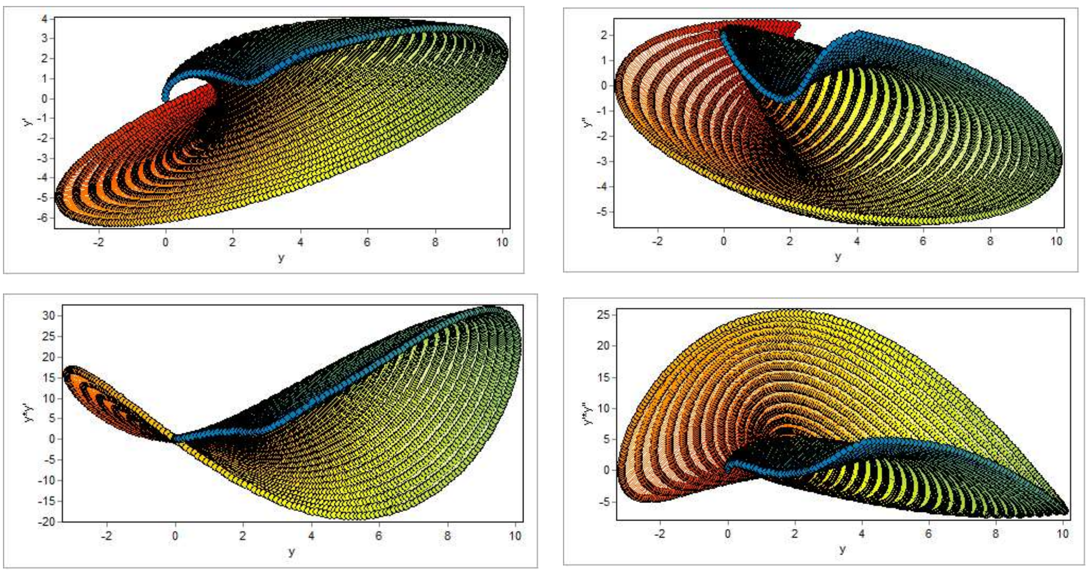


图 17.12 微分方程图-12

## 17.11 微分方程案例 - 11

微分方程：

$$\frac{d^2y}{dx^2} = 10 - \left(\frac{dy}{dx}\cos(x+a) + ay\right) \tag{17-13}$$

代码 17-13

```
LoopConstant a=[0:0.01:4];
Variable x=[0,4+a], y=a,y'=0.4^a;
Plot y[x],y',y'', (2*y'-y'');
ODEFunction y''=10-(y'*cos(x+a)+a*y);
```

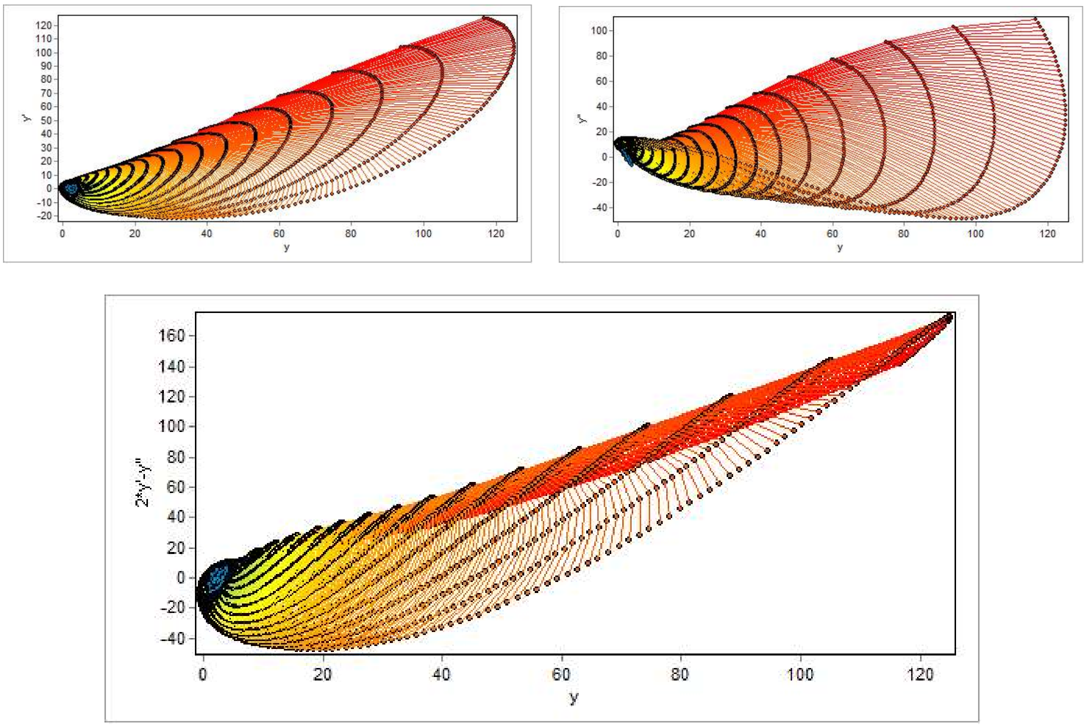



图 17.13 微分方程图-13

## 17.12 微分方程案例 - 12

微分方程组：

$$\begin{cases} \frac{dx}{dt} = cx(a_1x + a_2y + a_3) \\ \frac{dy}{dt} = y(b_1x + b_2y + b_3) \end{cases} \tag{17-14}$$

代码 17-14

```
LoopConstant c=[0.1:0.01:2];
Constant a(3)=[0,-1.5,1], b(3)=[2,0,-1.5];
Variable t=[0:0.02:10],x=1,y=1;
Plot x[x],y,x',y';
ODEFunction x'=c*x*(a1*x+a2*y+a3);
                    y'=y*(b1*x+b2*y+b3);
```

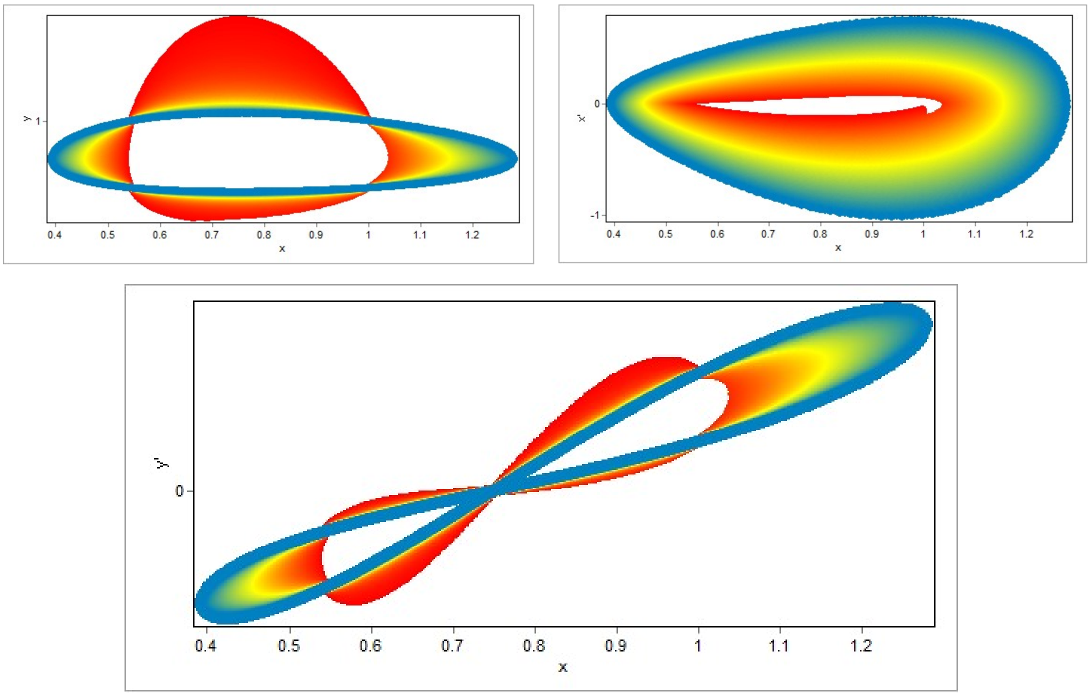



图 17.14 微分方程图-14

## 17.13 小结

多姿多彩、美妙绝伦，微分方程图之美令人叹为观止。牛顿和莱布尼茨两位科学巨匠，他们才是真正的幕后画师。