34. 经典工程优化问题之 1stOpt 求解

34.1 引言

解决实际工程优化问题是优化算法及优化求解器的重要目标和使命，也是数学理论与实际应用相结合的具体表现。在此选择了十五道经典的工程优化问题，其中大部分问题被广泛引用并用于优化算法的验证与比较，出现在诸多研究论文之中。1stOpt作为一款世界领先的优化求解器，对这些经典工程优化问题是否容易处理？求解效果又如何？

34.2 工程优化问题

34.2.1 焊接梁设计（Welded-beam design）

焊接梁设计问题是一个强约束最优化问题，其目的降低设计的制造成本。该优化问题可以描述为寻找满足切应力τ、弯曲应力σ、梁条弯曲载荷P_c、末端偏差δ和边界条件等约束的四个设计变量—即梁条的长度x_1、高度x_2、厚度x_3和焊缝厚度x_4，使得制造焊接梁的费用最小，参考图34.1，该问题是一个典型的非线性规划问题，其数学描述如下：

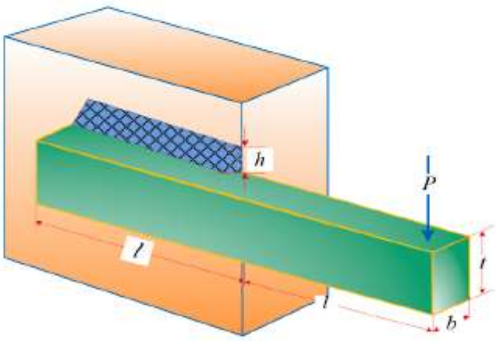

图34.1 焊接梁设计示意图

$$[h,l,t,b]=[x_1,x_2,x_3,x_4]$$

$$Min.\ \ 1.10471x_1^2x_2+0.04811x_3x_4(14.0+x_2)$$

$$s.t\begin{cases}\tau(x)-\tau_{max}\le 0\\ \sigma(x)-\sigma_{max}\le 0\\ x_1-x_4\le 0\\ 0.10471x_1^2+0.04811x_3x_4(14.0+x_2)-5.0\le 0\\ 0.125-x_1\le 0\\ \delta-\delta_{max}\le 0\\ p-p_c(x)\le 0\end{cases}$$

其中：

$$\tau(x)=\sqrt{(\tau')^2+2\tau'\tau''\frac{x_2}{2R}+(\tau'')^2}$$

$$\tau'=\frac{P}{\sqrt{2}x_1x_2},\quad \tau''=\frac{MR}{J},\quad M=P\left(L+\frac{x_2}{2}\right)$$

$$R=\sqrt{\frac{x_2^2}{4}+\left(\frac{x_1+x_3}{2}\right)^2},\quad J=2\left(\sqrt{2}x_1x_2\left(\frac{x_2^2}{12}+\left(\frac{x_1+x_3}{2}\right)^2\right)\right)$$

$$\sigma(x)=\frac{6PL}{x_3^2x_4},\quad \delta(x)=\frac{4PL^3}{Ex_3^3x_4},\quad p_c(x)=\frac{4.013E\sqrt{\frac{x_3^2x_4^6}{36}}}{L^2}P\left(1-\frac{x_3}{2L}\sqrt{\frac{E}{4G}}\right)$$

$$P=6000, L=14, E=3E+7, G=1.2E+7, \tau_{max}=13600, \sigma_{max}=30000, \delta_{max}=0.25$$

$$0.1\le x_1\le 2, 0.1\le x_{2,}x_3\le 10, 0.1\le x_4\le 2$$

代码：

```
Parameter x(4);
LowBound = [0.1,0.1,0.1,0.1];
UpBound = [2.0,10,10,2.0];
Constant P=6000,L=14,E=3e+7,G=1.2e+7,t_max=13600,s_max=30000,d_max=0.25;
ConstStr m = p*(l+x2/2),
         r = sqrt(x2^2/4+((x1+x3)/2)^2),
         j = 2*(sqrt(2)*x1*x2*(x2^2/12+((x1+x3)/2)^2)),
         p_c = (4.013*e*sqrt(x3^2*x4^6/36))/l^2*(1-x3/(2*l)*sqrt(e/(4*g))),
         t1 = p/(sqrt(2)*x1*x2), t2 = m*r/j,
         t = sqrt(t1^2+2*t1*t2*x2/(2*r)+t2^2),
         s = 6*p*l/(x3^2*x4),
         d = 4*p*l^3/(e*x3^3*x4);
MinFunction 1.10471*x1^2*x2+0.04811*x3*x4*(14.0+x2);
        t-t_max<=0;
        s-s_max<=0;
        x1-x4<=0;
        0.10471*x1^2+0.04811*x3*x4*(14.0+x2)-5.0<=0;
        0.125-x1<=0;
        d-d_max<=0;
        P-P_c<=0;
```

结果：

```
Objective Function (Min.): 1.72485230859741
x1: 0.205729639786082
x2: 3.47048866562805
x3: 9.03662391035747
x4: 0.205729639786087

Constrained Functions:
1: ((sqrt(((6000/(sqrt(2)*x1*x2)))^2+((6000/(sqrt(2)*x1*x2)))*((((6000*(14+x2/2)))*((sqrt(0.25*(x2^2+(x1+
x3)^2))))/((2*(sqrt(2)*x1*x2*(x2^2/12+0.25*(x1+x3)^2))))))*x2/((sqrt(0.25*(x2^2+(x1+x3)^2))))+((((6000*
(14+x2/2)))*((sqrt(0.25*(x2^2+(x1+x3)^2))))/((2*(sqrt(2)*x1*x2*(x2^2/12+0.25*(x1+x3)^2))))))^2)))-13600-
0 = -1.16415321826935E-10
2: ((6*6000*14/(x4*x3^2)))-30000-0 = -5.0931703299284E-11
3: x1-x4-0 = -5.49560397189452E-15
4: 0.10471*x1^2+0.04811*x3*x4*(14.0+x2)-5.0-0 = -3.43298378536221
5: 0.125-x1-0 = -0.0807296397860818
6: ((4*6000*14^3/(30000000*x4*x3^3)))-0.25-0 = -0.235540322584754
7: 6000-(((4.013*30000000/(6*14^2))*x3*x4^3*(1-0.25*x3*sqrt(30000000/12000000)/14)))-0 = -
6.12089934293181E-10
```

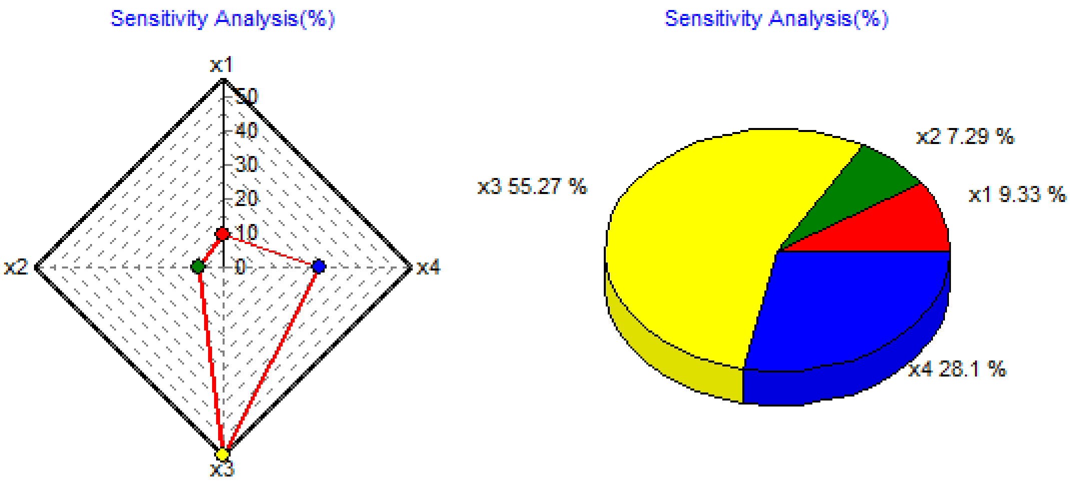

图34.2 焊接梁设计问题参数灵敏度分析示意图

34.2.2 压力容器设计问题（Pressure vessel design problem ）

压力容器设计问题是约束工程优化问题领域的经典案例之一，它的结构包括一个半球形封头盖和一个圆柱体，优化计算的目标是最小化生产总成本，包括材料成本、成形成本和焊接成本等。为了控制这些成本，在压力容器的设计过程中，需要选择一些参数，如壳体和封头的厚度（Ts=x1和Th=x2）、内半径（R=x3）和圆柱截面的长度（L=x4），以使其成本最小化。参考图34.3，该问题的数学描述如下：

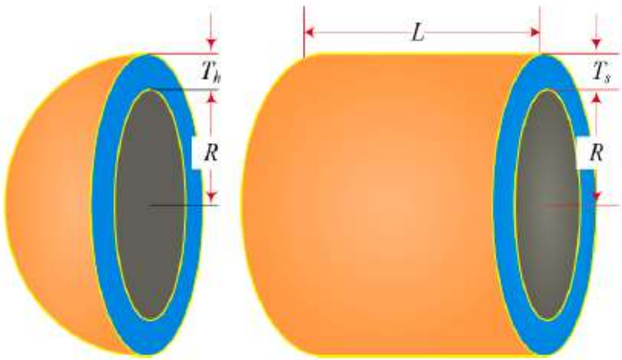

图34.3 焊接梁设计示意图

$$Min.\ \ 0.6224x_1x_3x_4 + 1.7781x_2x_3^2 + 3.1661x_1^2x_4 + 19.84x_1^2x_3$$

$$s.t.\begin{cases} -x_1 + 0.0193x_3 \le 0 \\ -x_2 + 0.00954x_3 \le 0 \\ -\pi x_3^2x_4 - \left(\dfrac{4}{3}\right)\pi x_3^3 + 1296000 \le 0 \\ x_4 - 240 \le 0 \end{cases}$$

其中：

$$1 \cdot 0.0625 \le x_1, x_2 \le 99 \cdot 0.0625, 10 \le x_3, x_4 \le 200$$

代码：

```
EnhancedBound = 1;
Parameter x(2)=[0.0625,99*0.0625],x(3:4)=[10,200];
MinFunction 0.6224*x1*x3*x4+1.7781*x2*x3^2+3.1661*x1^2*x4+19.84*x1^2*x3;
        -x1+0.0193*x3<=0;
        -x2+0.00954*x3<=0;
        -pi*x3^2*x4-(4/3)*pi*x3^3+1296000<=0;
        x4-240<=0;
```

结果：

```
Objective Function (Min.): 5885.33277361788
x1: 0.778168641375371
x2: 0.384649162628201
x3: 40.3196187241089
x4: 199.999999999858

Constrained Functions:
1: -x1+0.0193*x3-0 = -6.86117829218347E-14
2: -x2+0.00954*x3-0 = -2.02060590481778E-13
3: -pi*x3^2*x4-(4/3)*pi*x3^3+1296000-0 = 0
4: x4-240-0 = -40.0000000001423
```

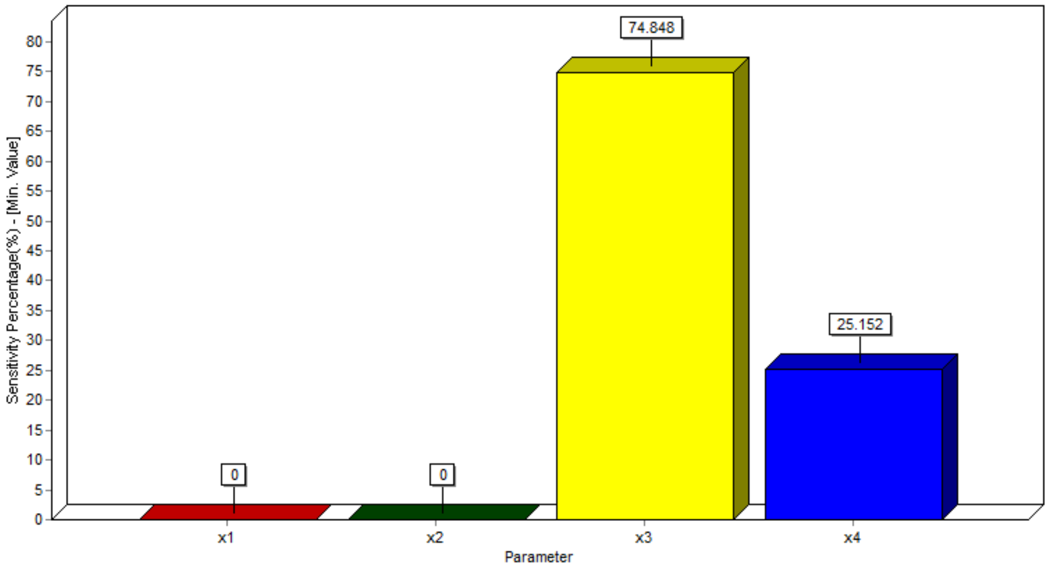

图34.4 压力容器设计问题参数灵敏度分析示意图

34.2.3 减速器设计问题（Speed Reducer Design）

减速器设计问题是一经典的约束工程优化问题，该问题的目标是使减速器的重量最小化。参考图34.5所示设计结构，减速器包含了2个安装了齿轮的独立轴，这2个轴通过轴承依次连接到主机架上。减速器设计优化问题包含11个不等式约束，7个计算参数包括：面宽 (x1)、齿模 (x2)、小齿轮齿数 (x3)、轴承间第一轴长度 (x4)、轴承间第二轴长度（x5）、第一轴直径（x6）以及第二轴的直径 (x7)，具体数学模型描述如下：

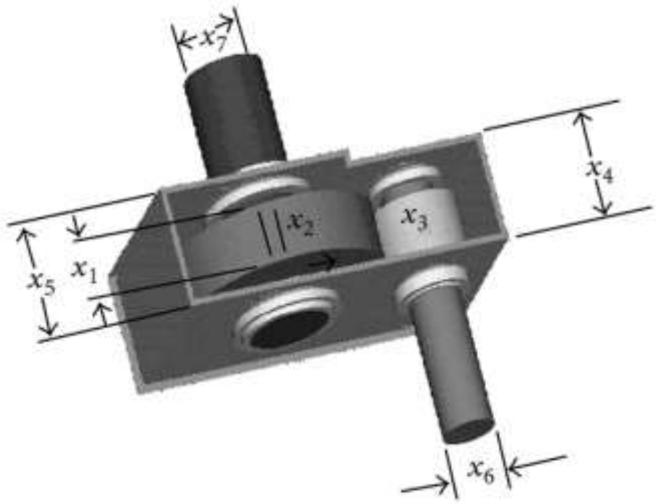

图34.5 减速器设计问题示意图

$$Min.\quad 0.7854x_1x_2^2(3.3333x_3^2+14.9334x_3-43.0934)-1.508x_1(x_6^2+x_7^2) \\ +7.4777(x_6^3+x_7^3)+0.7854(x_4x_6^2+x_5x_7^2)$$

$$s.t.\begin{cases}\dfrac{27}{x_1x_2^2x_3}-1\le 0\\ \dfrac{397.5}{x_1x_2^2x_3^2}-1\le 0\\ \dfrac{1.93x_4^3}{x_2x_3x_6^4}-1\le 0\\ \dfrac{1.93x_5^3}{x_2x_3x_7^4}-1\le 0\\ \dfrac{1}{110x_6^3}\sqrt{(\dfrac{745x_4}{x_2x_3})^2+16.910^6}-1\le 0\\ \dfrac{1}{85x_7^3}\sqrt{(\dfrac{745x_5}{x_2x_3})^2+157.510^6}-1\le 0\\ \dfrac{x_2x_3}{40}-1\le 0\\ \dfrac{5x_2}{x_1}-1\le 0\\ \dfrac{x_1}{12x_2}-1\le 0\\ \dfrac{1.5x_6+1.9}{x_4}-1\le 0\\ \dfrac{1.1x_7+1.9}{x_5}-1\le 0\end{cases}$$

其中：

$$2.6\le x_1\le 3.6, 0.7\le x_2\le 0.8, 17\le x_3\le 28$$
$$7.3\le x_4, x_5\le 8.3, 2.9\le x_6\le 3.9, 5\le x_7\le 5.5$$

代码：

```
Algorithm = DE2;
Parameter x1=[2.6,3.6],x2=[0.7,0.8],x3=[17,28],x4=[7.3,8.3],x5=[7.3,8.3],x6=[2.9,3.9],x7=[5.0,5.5];
MinFunction 0.7854*x1*x2^2*(3.3333*x3^2+14.9334*x3-43.0934)-
1.508*x1*(x6^2+x7^2)+7.4777*(x6^3+x7^3)+0.7854*(x4*x6^2+x5*x7^2);
        27/(x1*x2^2*x3)-1<=0;
        397.5/(x1*x2^2*x3^2)-1<=0;
        1.93*x4^3/(x2*x3*x6^4)-1<=0;
        1.93*x5^3/(x2*x3*x7^4)-1<=0;
        1/(110*x6^3)*sqrt((745*x4/(x2*x3))^2+16.9*10^6)-1<=0;
        1/(85*x7^3)*sqrt((745*x5/(x2*x3))^2+157.5*10^6)-1<=0;
        x2*x3/40-1<=0;
        5*x2/x1-1<=0;
        x1/(12*x2)-1<=0;
        (1.5*x6+1.9)/x4-1<=0;
        (1.1*x7+1.9)/x5-1<=0;
```

结果：

```
Objective Function (Min.): 2994.47106622968
x1: 3.5
x2: 0.7
x3: 17
x4: 7.3
x5: 7.71531991350076
x6: 3.35021466624566
x7: 5.28665446498091

Constrained Functions:
1: 27/(x1*x2^2*x3)-1-0 = -0.0739152803978735
2: 397.5/(x1*x2^2*x3^2)-1-0 = -0.197998527141949
3: 1.93*x4^3/(x2*x3*x6^4)-1-0 = -0.499172248191644
4: 1.93*x5^3/(x2*x3*x7^4)-1-0 = -0.904643904481127
5: 1/(110*x6^3)*sqrt((745*x4/(x2*x3))^2+16.9*10^6)-1-0 = -1.33615674080545E-10
6: 1/(85*x7^3)*sqrt((745*x5/(x2*x3))^2+157.5*10^6)-1-0 = -3.33066907387547E-16
7: x2*x3/40-1-0 = -0.7025
8: 5*x2/x1-1-0 = 0
9: x1/(12*x2)-1-0 = -0.583333333333333
10: (1.5*x6+1.9)/x4-1-0 = -0.0513257535111654
11: (1.1*x7+1.9)/x5-1-0 = -2.62045940502276E-10
```

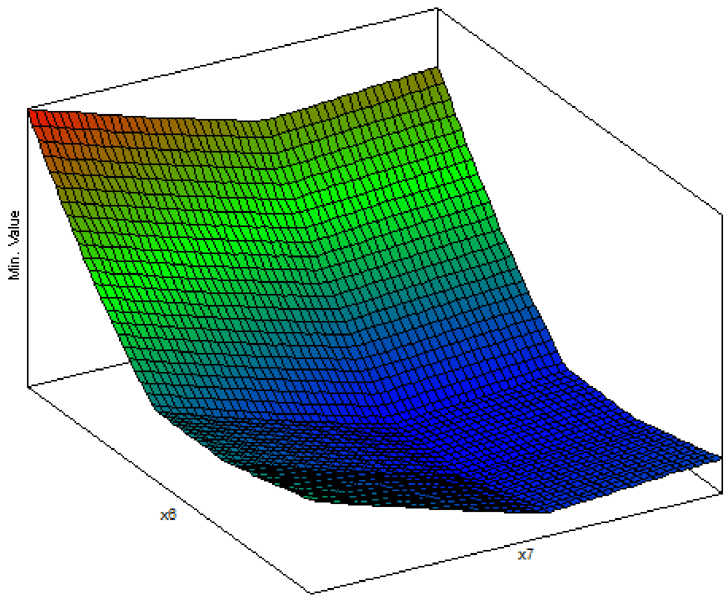

图34.6 减速器设计问题目标函数随参数x6及x7变化三维图

34.2.4 压缩弹簧设计问题（Compression Spring Design Problem）

压缩弹簧设计的目标是在满足一定约束条件下最小化其质量，其中包含最小挠度、剪切应力、振荡频率以及外径限制4个不等式约束，弹簧圈平均直径D(x2)、弹簧金属丝直径d(x1) 以及弹簧有效圈数P(x3)，如图34.7所示，其数学模型描述如下。

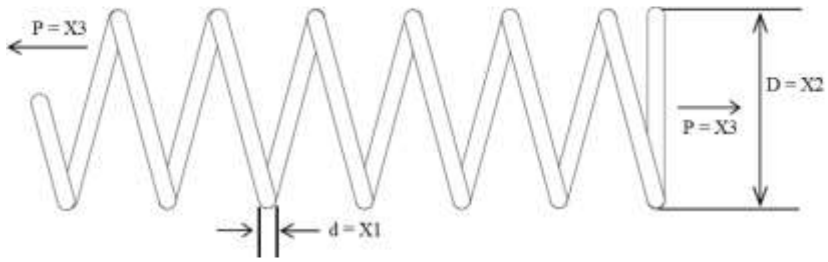

图34.7 压缩弹簧设计问题示意图

$$Min.\ \ (x_3+2)\cdot x_2\cdot x_1^2$$

$$s.t.\begin{cases}1-\dfrac{x_2^3x_3}{71785x_1^4}\le 0\\ \dfrac{4x_2^2-x_1x_2}{12566(x_2x_1^3-x_1^4)}+\dfrac{1}{5180x_1^2}-1\le 0\\ 1-\dfrac{140.45x_1}{x_2^2x_3}\le 0\\ \dfrac{x_2+x_1}{1.5}-1\le 0\end{cases}$$

其中：$0.05\le 2, 0.25\le x_2\le 1.3, 2\le x_3\le 15$

代码：

```
Parameter x1=[0.05,2.00],x2=[0.25,1.30],x3=[2.00,15.00];
MinFunction (x3+2)*x2*x1^2;
        1-x2^3*x3/(71785*x1^4)<=0;
        (4*x2^2-x1*x2)/(12566*(x2*x1^3-x1^4))+1/(5180*x1^2)-1<=0;
        1-140.45*x1/(x2^2*x3)<=0;
        (x2+x1)/1.5-1<=0;
```

结果：

```
Objective Function (Min.): 0.0126404011527912
x1: 0.0516337029570958
x2: 0.355838581129894
x3: 11.3242041742198

Constrained Functions:
1: 1-x2^3*x3/(71785*x1^4)-0 = -5.90860693705508E-13
2: (4*x2^2-x1*x2)/(12566*(x2*x1^3-x1^4))+1/(5180*x1^2)-1-0 = 0
3: 1-140.45*x1/(x2^2*x3)-0 = -4.05756248102787
4: (x2+x1)/1.5-1-0 = -0.728351810608673
```

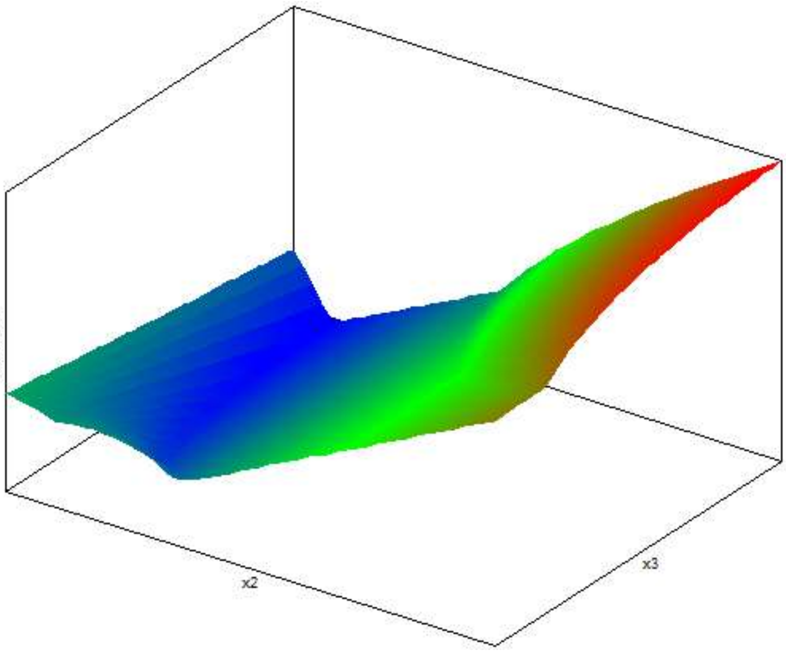

图34.8 压缩弹簧设计问题目标函数随参数x2及x3变化三维图

34.2.5 三杆桁架设计问题（Three-bar truss design problem）

三杆桁架设计是一个工程优化问题，其目标是求解最佳横截面积A1=A3=x1和A2=x2使静载桁架结构的体积最小，同时考虑到应力（σ）限制，如下图所示显示，其数学模型描述如下。

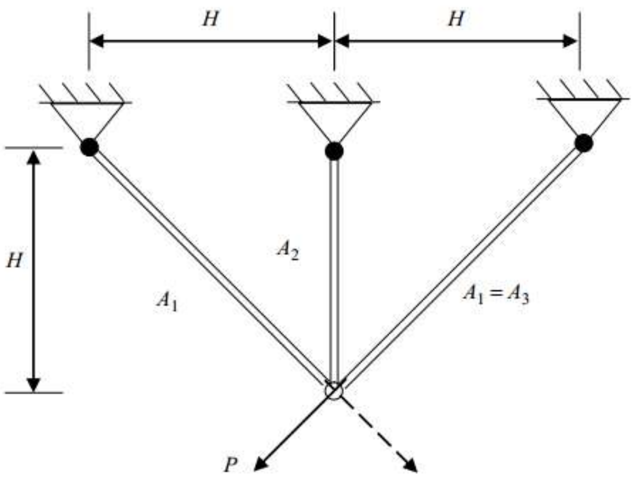

图34.9 三杆桁架设计问题示意图

$$[A_1, A_2] = [x_1, x_2]$$

$$Min.\ \left(2\sqrt{2}x_1 + x_2\right)H$$

$$s.t.\begin{cases}\dfrac{\sqrt{2}x_1+x_2}{\sqrt{2}x_12+2x_1x_2}P-\sigma\leq 0\\ \dfrac{x_2}{\sqrt{2}x_12+2x_1x_2}P-\sigma\leq 0\\ \dfrac{1}{x_1+\sqrt{2}x_2}P-\sigma\leq 0\end{cases}$$

其中：$0\leq x_1,x_2\leq 1,H=100,P=2,\sigma=2$

代码：

```
Constant H=100,P=2,c=2;
Parameter x(2)=[0,1];
MinFunction (2*sqrt(2)*x1 + x2)*H;
        (sqrt(2)*x1+x2)/(sqrt(2)*x1**2 + 2*x1*x2)*P-C<=0;
        x2/(sqrt(2)*x1**2 + 2*x1*x2)*P-C<=0;
        1/(x1+sqrt(2)*x2)*P-C<=0;
```

结果：

```
Objective Function (Min.): 263.895843376468
x1: 0.788675140591979
x2: 0.408248273501316

Constrained Functions:
1: (sqrt(2)*x1+x2)/(sqrt(2)*x1^2 + 2*x1*x2)*2-2-0 = -2.22044604925031E-16
2: x2/(sqrt(2)*x1^2 + 2*x1*x2)*2-2-0 = -1.46410163442098
3: 1/(x1+sqrt(2)*x2)*2-2-0 = -0.535898365579015
```

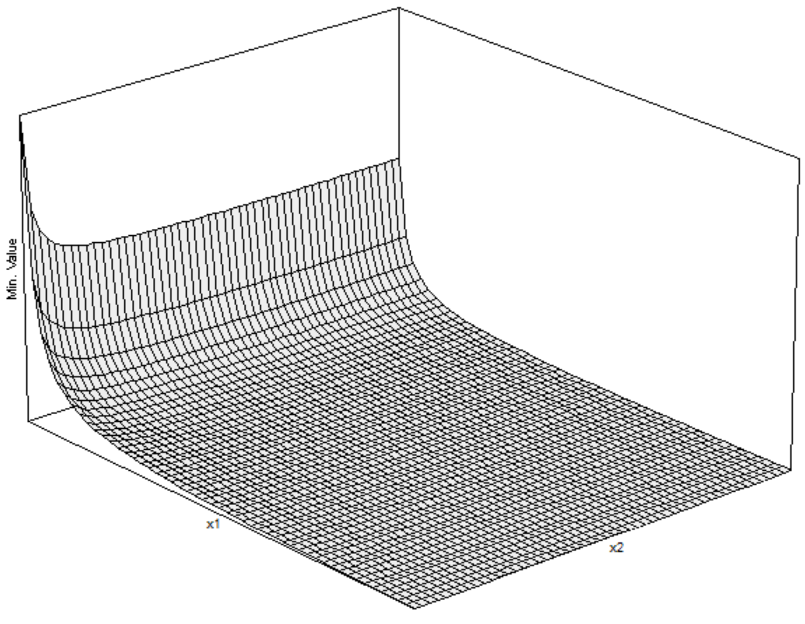

图34.10 三杆桁架设计问题目标函数随参数x1及x2变化三维图

34.2.6 五级悬臂梁设计问题（Five stage cantilever beam design）

五级悬臂梁设计问题，如下图示，在顶端承受载荷P的情况下，如何在满足约束的前提下使悬臂梁的体积最小。该问题共有十个几何变量和十一个约束，具体数学模型描述如下。

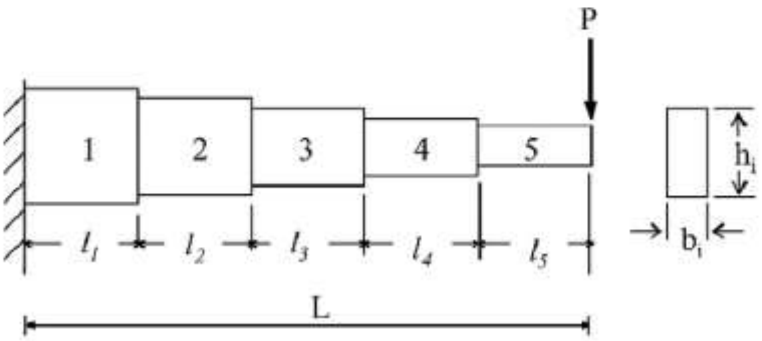

图34.11 五级悬臂梁设计问题示意图

$$Min.\ \sum_{i=1}^{5}(b_i h_i l_i)$$

$$s.t.\begin{cases}\frac{6Pl_5}{b_5h_5^2}-\sigma_{max}\leq 0\\ \frac{6P(l_5+l_4)}{b_4h_4^2}-\sigma_{max}\leq 0\\ \frac{6P(l_5+l_4+l_3)}{b_3h_3^2}-\sigma_{max}\leq 0\\ \frac{6P(l_5+l_4+l_3+l_2)}{b_2h_2^2}-\sigma_{max}\leq 0\\ \frac{6P(l_5+l_4+l_3+l_2+l_1)}{b_1h_1^2}-\sigma_{max}\leq 0\\ \frac{h_5}{b_5}-20\leq 0\\ \frac{h_4}{b_4}-20\leq 0\\ \frac{h_3}{b_3}-20\leq 0\\ \frac{h_2}{b_2}-20\leq 0\\ \frac{h_1}{b_1}-20\leq 0\\ \frac{PL^3}{3E}\left(\frac{244}{b_1h_1^3}+\frac{148}{b_2h_2^3}+\frac{76}{b_3h_3^3}+\frac{28}{b_4h_4^3}+\frac{4}{b_5h_5^3}\right)-\delta_{max}\leq 0\end{cases}$$

其中：$l_1=l_2=l_3=l_4=l_5=100,\sigma_{max}=14000,\delta_{max}=2.7,P=50000,E=2\cdot 10\text{^}7$

代码：

```
Algorithm = DE2;
Constant P=50000,E=2*10^7;
Constant [L1,L2,L3,L4,L5,LL]=100;
Parameter b1=[1,5],b(2:3)=[2.4,3.1],b(4:5)=[1,5],h1=[30,60],h(2:3)=[45,60],h(4:5)=[30,60];
MinFunction Sum(i=1:5)(b[i]*h[i]*L[i]);
        6*P*L5/(b5*h5^2)-14000<=0;
        6*P*(L5+L4)/(b4*h4^2)-14000<=0;
        6*P*(L5+L4+L3)/(b3*h3^2)-14000<=0;
        6*P*(L5+L4+L3+L2)/(b2*h2^2)-14000<=0;
```

6*P*(L5+L4+L3+L2+L1)/(b1*h1^2)-14000<=0;
h5/b5-20<=0;
h4/b4-20<=0;
h3/b3-20<=0;
h2/b2-20<=0;
h1/b1-20<=0;
P*LL^3/(3*E)*(244/(b1*h1^3)+148/(b2*h2^3)+76/(b3*h3^3)+28/(b4*h4^3)+4/(b5*h5^3))-2.7<=0;

结果：

Objective Function (Min.): 61914.7890088845
b1: 2.99204240294288
b2: 2.77756612143912
b3: 2.52358629860996
b4: 2.20455569154185
b5: 1.74975701193801
h1: 59.8408480588575
h2: 55.5513224287824
h3: 50.4717259721992
h4: 44.0911138308369
h5: 34.9951402387601

Constrained Functions:
1: 6*50000*100/(b5*h5^2)-14000-0 = -1.81898940354586E-12
2: 6*50000*(100+100)/(b4*h4^2)-14000-0 = -1.2732925824821E-11
3: 6*50000*(100+100+100)/(b3*h3^2)-14000-0 = 0
4: 6*50000*(100+100+100+100)/(b2*h2^2)-14000-0 = -1.81898940354586E-12
5: 6*50000*(100+100+100+100+100)/(b1*h1^2)-14000-0 = 0
6: h5/b5-20-0 = -4.61852778244065E-14
7: h4/b4-20-0 = 0
8: h3/b3-20-0 = -1.4210854715202E-14
9: h2/b2-20-0 = 0
10: h1/b1-20-0 = -7.105427357601E-15
11: 50000*100^3/(3*20000000)*(244/(b1*h1^3)+148/(b2*h2^3)+76/(b3*h3^3)+28/(b4*h4^3)+4/(b5*h5^3))-2.7-0 = -1.76071500002789

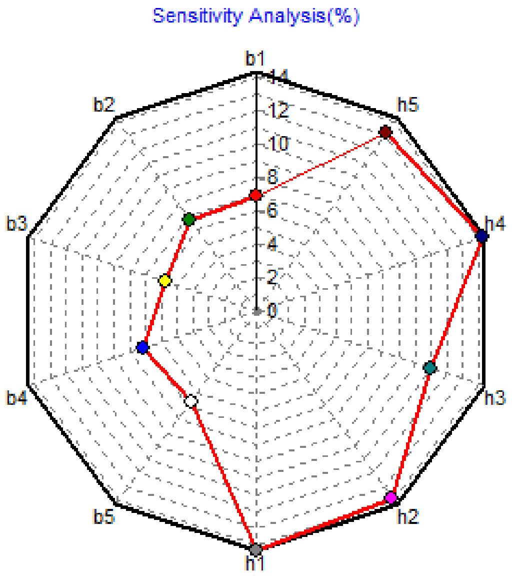

图34.12 五级悬臂梁设计问题参数灵敏度分析示意图

34.2.7 三级换热器设计（Three stage heat exchanger design）

如图34.13示，对三级串联换热器进行设计优化设计，其目标是最小化传热面积之和，

该问题共有八个设计参数和六个不等式约束（三个线性约束和三个非线性约束），其数学模型描述如下。

图34.13 三级换热器设计问题示意图

$$[A_1, A_2, A_3, T_1, T_2, t_{11}, t_{21}, t_{31}] = [x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8]$$

$$Min. \ \ x_1 + x_2 + x_3$$

$$s.t. \begin{cases} x_4 + x_6 - t_{11} - T_0 \leq 0 \\ x_5 + x_7 - t_{21} - x_4 \leq 0 \\ t_3 + x_8 - t_{31} - x_5 \leq 0 \\ x_4 + \dfrac{U_1}{w} x_1 T_0 - T_0 - \dfrac{U_1}{w} x_1 x_6 \leq 0 \\ x_5 + \dfrac{U_2}{w} x_2 x_4 - x_4 - \dfrac{U_2}{w} x_2 x_7 \leq 0 \\ T_3 + \dfrac{U_3}{w} x_3 x_5 - x_5 - \dfrac{U_3}{w} x_3 x_8 \leq 0 \end{cases}$$

其中：$T_0 = 100, T_3 = 500, w = 10^5, t_{11} = 300, t_{21} = 400, t_{31} = 600, U_1 = 120, U_2 = 80, U_3 = 40$

代码：

```
Algorithm = DE1;
Constant T0=100,T3=500,w=10^5,t11=300,t21=400,t31=600,U1=120,U2=80,U3=40;
Parameter x(8)>0;
MinFunction x1+x2+x3;
                x4+x6-t11-T0<=0;
                x5+x7-t21-x4<=0;
                T3+x8-t31-x5<=0;
                x4+U1/w*x1*t0-T0-U1/w*x1*x6<=0;
                x5+U2/w*x2*x4-x4-U2/w*x2*x7<=0;
                T3+U3/w*x3*x5-x5-U3/w*x3*x8<=0;
```

结果：

```
Objective Function (Min.): 7049.24927247623
x1: 579.306727517072
x2: 1359.97146450105
x3: 5109.9710804581
x4: 182.01759861862
x5: 295.601156781679
x6: 217.98240138138
x7: 286.41644183694
x8: 395.601156781678

Constrained Functions:
1: x4+x6-300-100-0 = -5.6843418860808E-14
2: x5+x7-400-x4-0 = -1.4210854715202E-12
3: 500+x8-600-x5-0 = -1.36424205265939E-12
4: x4+120/100000*x1*100-100-120/100000*x1*x6-0 = -2.78532752417959E-12
5: x5+80/100000*x2*x4-x4-80/100000*x2*x7-0 = -8.92441676114686E-12
6: 500+40/100000*x3*x5-x5-40/100000*x3*x8-0 = 0
```

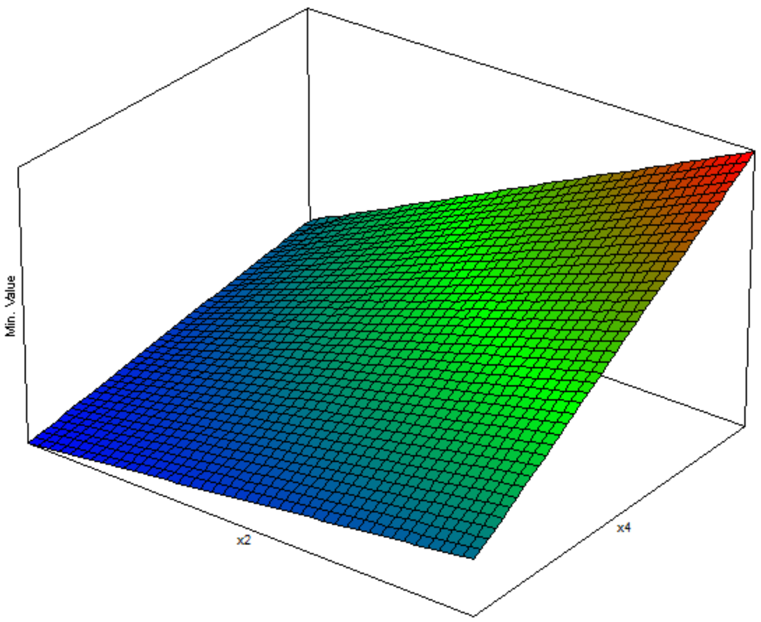

图34.14三级换热器设计问题目标函数随参数x2及x4变化三维图

34.2.8 汽车侧碰设计问题（Car side impact design problem）

汽车侧面碰撞设计问题是优化计算出9个影响参数使得车门重量最小，这9个参数包括B柱内板的厚度(x1)、B柱加强件(x2)、地板内侧厚度(x3)，横梁(x4)、门梁(x5)、门带线加强件(x6)、车顶纵梁(x7)，B柱内侧(x8)、地板内侧(x9)、护栏高度(x10)和碰撞位置(x11)，参考图34.15，其数学模型描述如下：

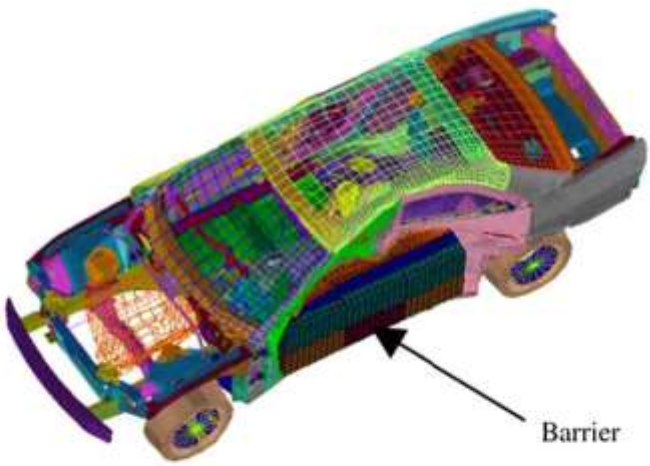

图34.15 汽车侧碰设计问题示意图

$$Min.\ \ 1.98 + 4.90x_1 + 6.67x_2 + 6.98x_3 + 4.01x_4 + 1.78x_5 + 2.73x_7$$

$$
s.t.\begin{cases}
1.16-0.3717x_2x_4-0.00931x_2x_{10}-0.484x_3x_9+0.01343x_6x_{10}\le 1\\
46.36-9.9x_2-12.9x_1x_8+0.1107x_3x_{10}\le 32\\
33.86+2.95x_3+0.1792x_{10}-5.057x_1x_2-11.0x_2x_8-\\
\qquad 0.0215x_5x_{10}-9.98x_7x_8+22.0x_8x_9\le 32\\
28.98+3.818x_3-4.2x_1x_2+0.0207x_5x_{10}+6.63x_6x_9-\\
\qquad 7.7x_7x_8+0.32x_9x_{10}\le 32\\
0.261-0.0159x_1x_2-0.188x_1x_8-0.019x_2x_7+0.0144\cdot x_3x_5+\\
\qquad 0.0008757x_5x_{10}+0.08045x_6x_9+0.00139x_8x_{11}+\\
\qquad 0.00001575x_{10}x_{11}\le 0.32\\
0.214+0.00817x_5-0.131x_1x_8-0.0704x_1x_9+0.03099x_2x_6-\\
\qquad 0.018x_2x_7+0.0208x_3x_8+0.121x_3x_9-0.00364x_5x_6+\\
\qquad 0.0007715x_5x_{10}-0.0005354x_6x_{10}+0.00121x_8x_{11}\le 0.32\\
0.74-0.061x_2-0.163x_3x_8+0.001232x_3x_{10}-0.166x_7x_9\\
\qquad +0.227x_2^2\le 0.32\\
4.72-0.5x_4-0.19x_2x_3-0.0122x_4x_{10}+0.009325x_6x_{10}\\
\qquad +0.000191x_{11}^2\le 4\\
10.58-0.674x_1x_2-1.95x_2x_8+0.02054x_3x_{10}-0.0198x_4x_{10}\\
\qquad +0.028x_6x_{10}\le 9.9\\
16.45-0.489x_3x_7-0.843x_5x_6+0.0432x_9x_{10}-0.0556x_9x_{11}\\
\qquad -0.000786x_{11}^2\le 15.7
\end{cases}
$$

其中：$0.5\le x_1 \dots x_7\le 1.5, 0\le x_8, x_9\le 1, -30\le x_{10}, x_{11}\le 30$

代码：

```
Algorithm = DE2;
Parameter x(7)=[0.5,1.5], x(8:9)=[0,1],x(10:11)=[-30,30];
MinFunction 1.98+4.90*x1+6.67*x2+6.98*x3+4.01*x4+1.78*x5+2.73*x7;
        1.16-0.3717*x2*x4-0.00931*x2*x10-0.484*x3*x9+0.01343*x6*x10<=1;
        46.36-9.9*x2-12.9*x1*x8+0.1107*x3*x10<=32;
        33.86+2.95*x3+0.1792*x10-5.057*x1*x2-11.0*x2*x8-0.0215*x5*x10-
9.98*x7*x8+22.0*x8*x9<=32;
        28.98+3.818*x3-4.2*x1*x2+0.0207*x5*x10+6.63*x6*x9-7.7*x7*x8+0.32*x9*x10<=32;
        0.261-0.0159*x1*x2-0.188*x1*x8-
0.019*x2*x7+0.0144*x3*x5+0.0008757*x5*x10+0.08045*x6*x9+0.00139*x8*x11+0.00001575*x10*x11<=0
.32;
        0.214+0.00817*x5-0.131*x1*x8-0.0704*x1*x9+0.03099*x2*x6-
0.018*x2*x7+0.0208*x3*x8+0.121*x3*x9-0.00364*x5*x6+0.0007715*x5*x10-
0.0005354*x6*x10+0.00121*x8*x11<=0.32;
        0.74-0.061*x2-0.163*x3*x8+0.001232*x3*x10-0.166*x7*x9+0.227*x2^2<=0.32;
        4.72-0.5*x4-0.19*x2*x3-0.0122*x4*x10+0.009325*x6*x10+0.000191*x11^2<=4;
        10.58-0.674*x1*x2-1.95*x2*x8+0.02054*x3*x10-0.0198*x4*x10+0.028*x6*x10<=9.9;
        16.45-0.489*x3*x7-0.843*x5*x6+0.0432*x9*x10-0.0556*x9*x11-0.000786*x11^2<=15.7;
```

结果：

```
Objective Function (Min.): 25.7423722959119
x1: 0.500000000000416
x2: 0.500002016807817
x3: 1.03031211611582
x4: 1.44657862177387
x5: 0.500000000000041
x6: 1.4999999999991
x7: 1.5
x8: 0.999999999999956
x9: 0.976825867328526
x10: -27.6361682361755
x11: -1.28274767484969E-5
```

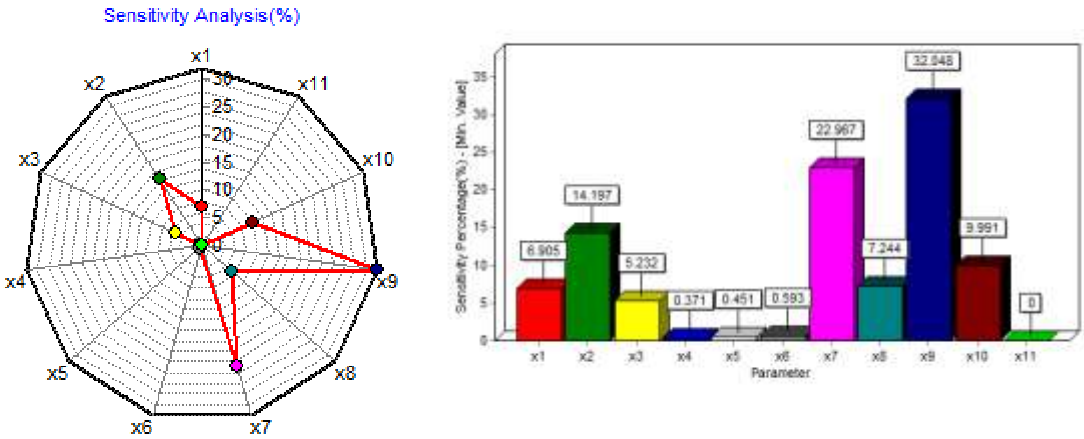

图34.16汽车侧碰设计问题参数灵敏度分析示意图

34.2.9 工字梁优化设计问题(Optimal Design of I-Shaped Beam)

工字梁优化设计问题，其目标是找到在满足预设载荷下横截面积和应力约束的同时，使工字梁的垂直挠度最小化的最佳变量。向量x = (x1,x2,x3,x4) = (h,b,tw,tf)，数学模型描述如下：

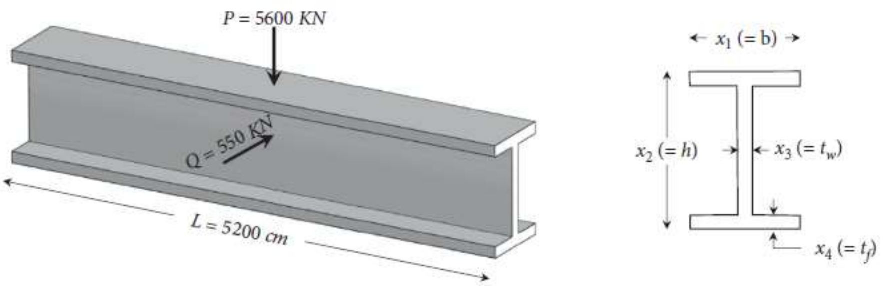

图34.17 工字梁优化设计问题示意图

$$[h, b, t_w, t_f] = [x_1, x_2, x_3, x_4]$$

$$Min.\ \frac{5000}{\frac{x_3(x_1-2x_4)^3}{12}+\frac{x_2x_4^3}{6}+2x_2x_4\left(\frac{x_1-x_4}{2}\right)^2}$$

$$s.t.\begin{cases}2x_2x_3+x_3(x_1-2x_4)\le 300\\ \frac{18x_1 10^4}{x_3(x_1-2x_4)^3+2x_2x_3\left(4x_4^2+3x_1(x_1-2x_4)\right)}+\frac{15x_2 10^3}{(x_1-2x_4)x_3^2+2x_3x_2^3}\le 56\end{cases}$$

其中：$10 \le x_1 \le 50, 10 \le x_2 \le 80, 0.9 \le x_3, x_4 \le 5$

代码：

```
Algorithm = DE2;
Parameter x1=[10,80], x2=[10,50], x(3:4)=[0.9,5];
MinFunction 5000/(x3*(x1-2*x4)^3/12+(x2*x4^3/6)+2*x2*x4*((x1-x4)/2)^2);
          2*x2*x3+x3*(x1-2*x4)<=300;
          18*x1*10^4/(x3*(x1-2*x4)^3+2*x2*x3*(4*x4^2+3*x1*(x1-2*x4)))+15*x2*10^3/((x1-
2*x4)*x3^2+2*x3*x2^3)<=56;
```

结果：

```
Objective Function (Min.): 0.00662595816551903
x1: 80
x2: 50
x3: 1.76470588235294
x4: 5

Constrained Functions:
1: 2*x2*x3+x3*(x1-2*x4)-(300) = -1.13686837721616E-13
2: 18*x1*10^4/(x3*(x1-2*x4)^3+2*x2*x3*(4*x4^2+3*x1*(x1-2*x4)))+15*x2*10^3/((x1-2*x4)*x3^2+2*x3*x2^3)-(56) = -50.2870668355151
```

34.2.10 管柱设计问题（Tubular Column Design）

该问题的目标是使用最小成本来获得一个均匀的管柱，该柱可以承受P=2500 kgf的压缩载荷。平均直径d和厚度t在[2,14]和[0.2,0.8]的范围内变化。柱组成材料中的特征参数为：屈服应力 σ = 500 kgf/cm^2、弹性模量E = 0.85×10^6 kgf/cm^{2}和密度 ρ = 0.0025 kgf/cm ^{3}。柱子的长度L为250 cm，向量x = (x1,x2) = (d,t)，数学模型描述如下：

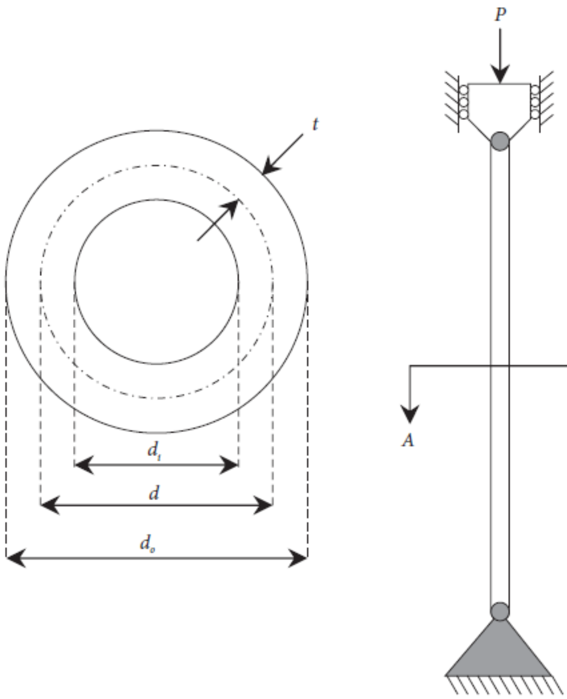

图34.18 管柱设计问题示意图

$$Min.\ \ 9.8x_1x_2+2x_1$$

$$s.t.\begin{cases}\dfrac{P}{\pi x_1x_2\sigma}-1\le 0\\ \dfrac{8PL^2}{\pi^3Ex_1x_2(x_1^2+x_2^2)}-1\le 0\end{cases}$$

其中：$2\le x_1\le 14, 0.2\le x_2\le 0.8,\ P=2500,\ \sigma=500, E=0.85E+6, L=250$

代码：

```
Constant P=2500,Sigma=500, E=0.85E+6, L=250;
Parameter x1=[2,14],x2=[0.2,0.8];
MinFunction 9.8*x1*x2+2*x1;
                  P/(pi*x1*x2*Sigma)-1<=0;
                  8*P*L^2/(pi^3*E*x1*x2*(x1^2+x2^2))-1<=0;
```

结果：

```
Objective Function (Min.): 26.4994968915155
x1: 5.45115623425486
x2: 0.291965477143678

Constrained Functions:
1: 2500/(pi*x1*x2*500)-1-0 = 0
2: 8*2500*250^2/(pi^3*850000*x1*x2*(x1^2+x2^2))-1-0 = -3.33066907387547E-16
```

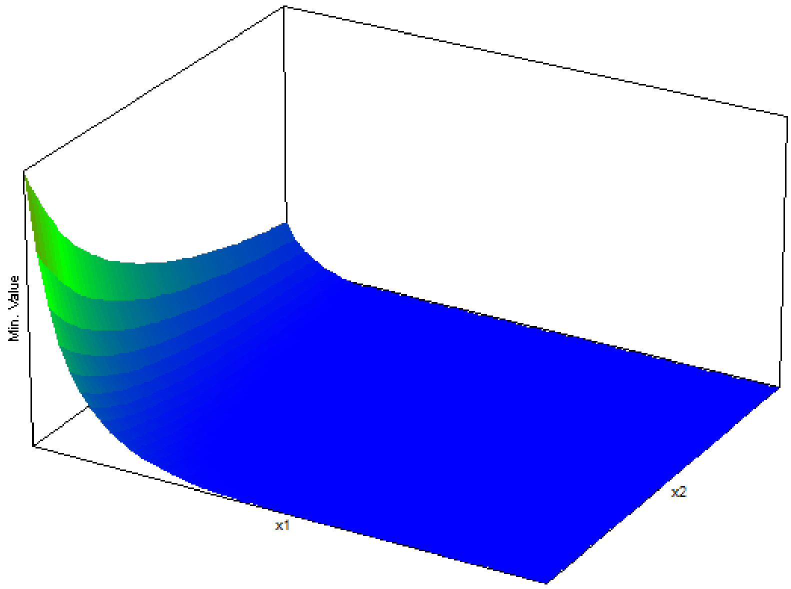

图34.19 管柱设计问题目标函数随参数x1及x2变化三维图

34.2.11 活塞杆优化问题 (Piston Lever Problem)

该问题的主要目的是通过将活塞杆从0°提升至45°时的油量降至最低时确定活塞杆部件H (x1) , B (x2) , D (x3)和H (x4)。该问题的数学公式描述如下。

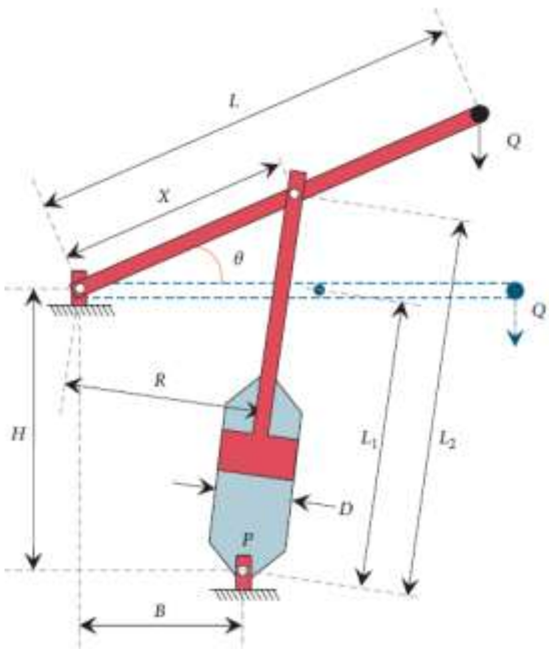

图34.20 活塞杆优化问题示意图

$$[H, B, X, D] = [x_1, x_2, x_3, x_4]$$

$$Min.\ \ \frac{1}{4}\pi x_4^2(L_2 - L_1)$$

$$\begin{cases} QL\cos(\theta) - RF \le 0 \\ Q(L - x_3) - M_{max} \le 0 \\ 1.2(L_2 - L_1) - L_1 \le 0 \\ \dfrac{x_4}{2} - x_2 \le 0 \end{cases}$$

其中：

$$R = \frac{\left|-x_3(x_3\sin(\theta) + x_1) + x_1\left(x_2 - x_3\cos(\theta)\right)\right|}{\sqrt{(x_3 - x_2)^2 + x_1^2}}$$

$$F = \frac{\pi P x_4^2}{4}$$

$$L_1 = \sqrt{(x_3 - x_2)^2 + x_1^2}$$

$$L_2 = \sqrt{(x_3\sin(\theta) + x_1)^2 + (x_2 - x_3\cos(\theta))^2}$$

$$\theta = 45\pi/180, Q = 10000, L = 240, Mmax = 1.8 * 10\text{^}6, P = 1500$$

$$0.05 \le x_1, x_2, x_4 \le 500, 0.05 \le x_3 \le 120$$

代码：

```
Constant theta=45*pi/180,Q=10000,L=240,Mmax=1.8*10^6,P=1500;
ConstStr R=abs(-x3*(x3*sin(theta)+x1)+x1*(x2-x3*cos(theta)))/sqrt((x3-x2)^2+x1^2),
         F=pi*P*x4^2/4,
         L1=sqrt((x3-x2)^2+x1^2),
         L2=sqrt((x3*sin(theta)+x1)^2+(x2-x3*cos(theta))^2);
Parameter x(4);
Algorithm = SM2;
```

```
UpBound = [500,500,120,500];
LowBound = [0.05,0.05,0.05,0.05];
MinFunction 1/4*pi*x4^2*(L2-L1);
                    Q*L*cos(theta)-R*F<=0;
                    Q*(L-x3)-Mmax<=0;
                    1.2*(L2-L1)-L1<=0;
                    x4/2-x2<=0;
```

结果：

```
Objective Function (Min.): 8.41269832310667
x1: 0.050000000000012
x2: 2.04151358991813
x3: 120
x4: 4.08302717983623

Constrained Functions:
1: 10000*240*cos(0.785398163397448)-((abs(-x3*(x3*sin(0.785398163397448)+x1)+x1*(x2-
x3*cos(0.785398163397448)))/
sqrt((x3-x2)^2+x1^2)))*((pi*1500*x4^2/4))-0 = -1.86264514923096E-9
2: 10000*(240-x3)-1800000-0 = -599999.999999996
3: 1.2*(((sqrt((x3*sin(0.785398163397448)+x1)^2+(x2-x3*cos(0.785398163397448))^2)))-((sqrt((x3-
x2)^2+x1^2))))-
((sqrt((x3-x2)^2+x1^2)))-0 = -117.18748323014
4: x4/2-x2-0 = -1.37667655053519E-14
```

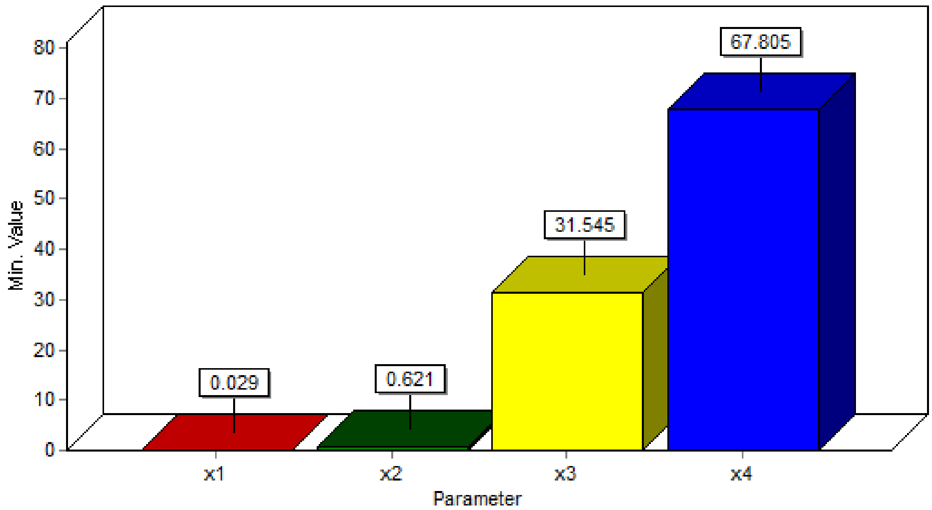

图34.21活塞杆优化问题参数灵敏度分析示意图

34.2.12 钢筋混凝土梁设计问题（Reinforced concrete beam）

假定钢筋混凝土梁的跨度为30ft，并承受2000 lbf的活荷载和1000 lbf的恒荷载，包括梁的重量。混凝土抗压强度(σ_c)为5 ksi、钢筋屈服应力(σ_y)为50 ksi。混凝土成本为$0.02/in2/linear ft，钢材成本为$1.0/in2/linear ft。为了使结构的总成本最小化，必须确定钢筋的面积As (x1)、梁的宽度b (x2)以及梁的深度h (x3)，梁的深宽比限制为小于或等于4。该优化问题可以描述如下：

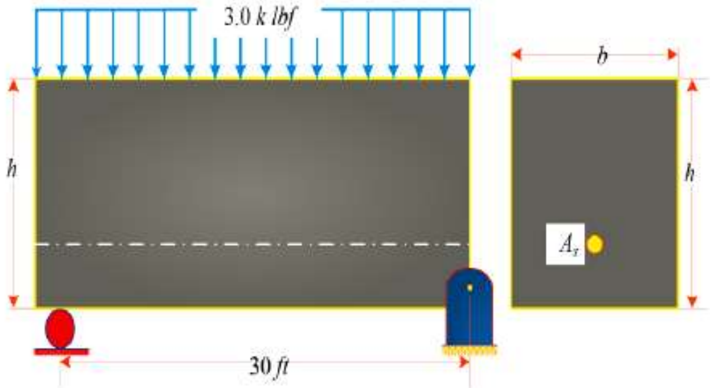

图34.22 钢筋混凝土梁设计问题

$$[A_s, b, h] = [x_1, x_2, x_3]$$

$$Min.\ \ 2.9x_1 + 0.6x_2x_3$$

$$s.t.\begin{cases} \dfrac{x_2}{x_3} - 4 \le 0 \\ 180 + \dfrac{7.375x_1^2}{x_3} - x_1x_2 \le 0 \end{cases}$$

其中：$1 \le x_1 \le 28, 28 \le x_2 \le 40, 5 \le x_3, x_4 \le 10$

代码：

```
Parameter x1=[1,28],x2=[28,40],x3=[5,10];
MinFunction 2.9*x1+0.6*x2*x3;
            x2/x3-4<=0;
            180+7.375*x1^2/x3-x1*x2<=0;
```

结果：

```
Objective Function (Min.): 149.190085217104
x1: 10.8193202576482
x2: 28.0254713275768
x3: 7.0063678318942

Constrained Functions:
1: x2/x3-4-0 = 0
2: 180+7.375*x1^2/x3-x1*x2-0 = 0
```

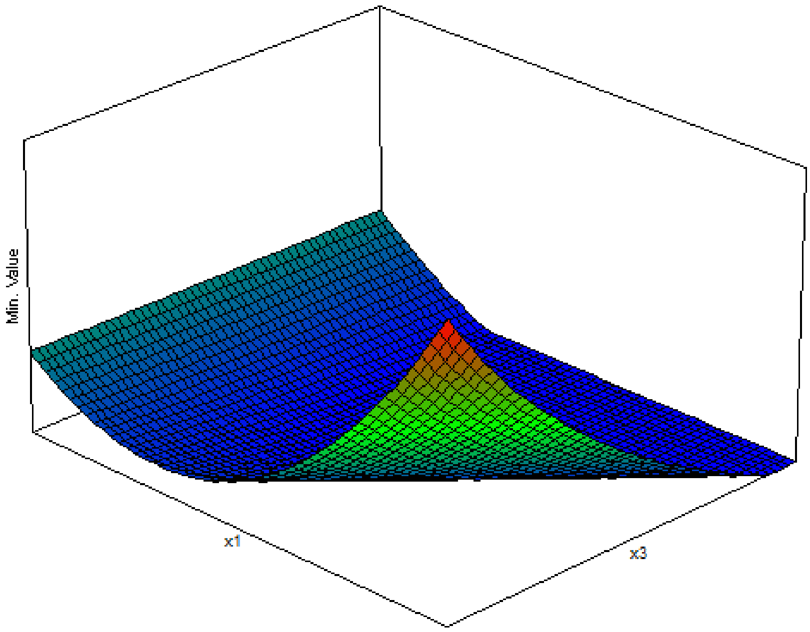

图34.23钢筋混凝土梁设计问题目标函数随参数x1及x3变化三维图

34.2.13 槽形舱壁设计问题(Corrugated Bulkhead Design)

槽形舱壁设计问题目的是将化学品罐车波槽形舱壁的重量最小化，其设计变量为宽度(x1)、深度(x2)、长度(x3)和板厚(x4)，该优化问题数学模型如下。

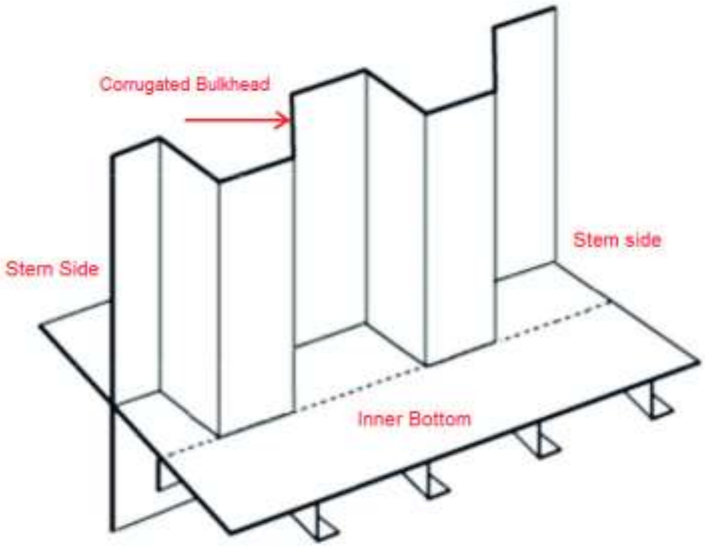

图34.24 槽形舱壁设计问题

$$Min.\quad \frac{5.885x_4(x_1+x_3)}{x_1+\sqrt{|x_3^2-x_2^2|}}$$

$$s.t.\begin{cases} -x_4x_2\left(0.4x_1+\dfrac{x_3}{6}\right)+8.94\left(x_1+\sqrt{|x_3^2-x_2^2|}\right)\le 0 \\ -x_4x_2^2\left(0.2x_1+\dfrac{x_3}{12}\right)+2.2\left(8.94\left(x_1+\sqrt{|x_3^2-x_2^2|}\right)\right)^{\frac{4}{3}}\le 0 \\ -x_4+0.0156x_1+0.15\le 0 \\ -x_4+0.0156x_3+0.15\le 0 \\ -x_4+1.05\le 0 \\ -x_3+x_2\le 0 \end{cases}$$

其中：$0\le x_1,x_2,x_3\le 100, 0\le x_4\le 5$

代码：

```
Parameter x(3)=[0,100],x4=[0,5];
MinFunction 5.885*x4*(x1+x3)/(x1+sqrt(abs(x3^2-x2^2)));
-x4*x2*(0.4*x1+x3/6)+8.94*(x1+sqrt(abs(x3^2-x2^2)))<=0;
-x4*x2^2*(0.2*x1+x3/12)+2.2*(8.94*(x1+sqrt(abs(x3^2-x2^2))))^(4/3)<=0;
-x4+0.0156*x1+0.15<=0;
-x4+0.0156*x3+0.15<=0;
-x4+1.05<=0;
-x3+x2<=0;
```

结果-1：

```
Objective Function (Min.): 6.84295801008077
x1: 57.6923076923077
x2: 34.1476203486744
x3: 57.6923076923077
x4: 1.05

Constrained Functions:
1: -x4*x2*(0.4*x1+x3/6)+8.94*(x1+sqrt(abs(x3^2-x2^2)))-0 = -240.694624799969
2: -x4*x2^2*(0.2*x1+x3/12)+2.2*(8.94*(x1+sqrt(abs(x3^2-x2^2))))^(4/3)-0 = -1.81898940354586E-11
3: -x4+0.0156*x1+0.15-0 = -2.4980018054066E-16
4: -x4+0.0156*x3+0.15-0 = -5.82867087928207E-16
5: -x4+1.05-0 = 0
6: -x3+x2-0 = -23.5446873436332
```

结果-2：

```
Objective Function (Min.): 0
x1: 0
x2: 1.57172778470263E-162
x3: 0
x4: 2.52354302910367

Constrained Functions:
1: -x4*x2*(0.4*x1+x3/6)+8.94*(x1+sqrt(abs(x3^2-x2^2)))-0 = 1.98714632203966E-161
2: -x4*x2^2*(0.2*x1+x3/12)+2.2*(8.94*(x1+sqrt(abs(x3^2-x2^2))))^(4/3)-0 = 1.1841202562503E-214
3: -x4+0.0156*x1+0.15-0 = -2.37354302910367
4: -x4+0.0156*x3+0.15-0 = -2.37354302910367
5: -x4+1.05-0 = -1.47354302910367
6: -x3+x2-0 = 1.57172778470263E-162
```

该问题用1stOpt求解可以得到上述两组结果，虽然第二组结果明显不符合实际，但从数学角度看却是准确无误的，符合所有参数范围限制并满足所有约束条件，该问题出现的原因就是参数x1、x2和x3的范围限制下限值为大于等于0，将该下限制改为一个略大于0的值如1E-6，就能避免出现正确但不符合实际的结果。

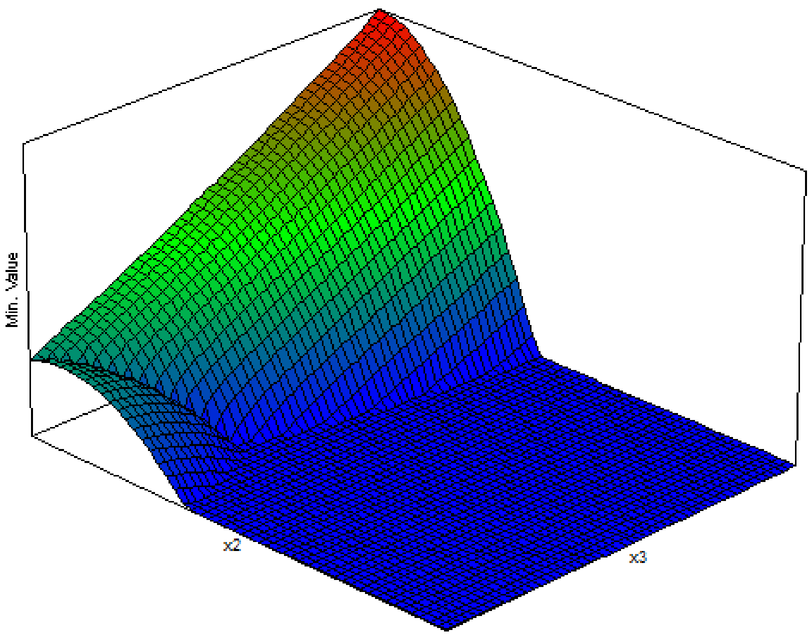

图34.25槽形舱壁设计问题目标函数随参数x2及x3变化三维图

34.2.14 齿轮系设计问题(Design of Gear Train)

齿轮系优化设计问题是机械工程中的一个无约束离散整数设计问题，其目标是最小化齿轮比，齿轮比定义为输出轴角速度与输入轴角速度之比。将齿轮的齿数A (x1)，B(x2), C (x3)和D(x4)视为设计变量，其数学模型如下：

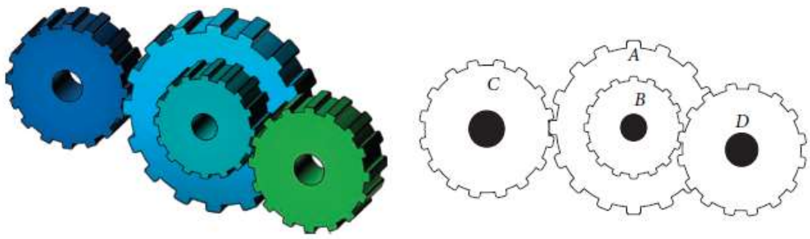

图34.26 齿轮系设计问题示意图

$$Min.\ \left(\frac{1}{6.931}-\frac{x_3x_2}{x_1x_4}\right)^2$$

其中：$16 \le x_1, x_2, x_3, x_4 \le 60$且均为整数。

代码：

```
Algorithm = SM2;
IntParameter x(4)=[12,60];
MinFunction (1/6.931-x3*x2/(x1*x4))^2;
```

结果：

```
Objective Function (Min.): 2.70085714888651E-12
x1: 49
x2: 16
```

x3: 19
x4: 43

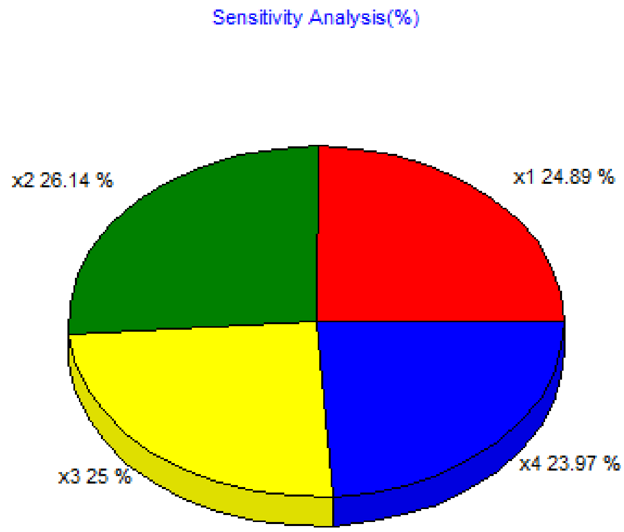

图 34.27 齿轮系设计问题参数灵敏度分析示意图

34.2.15 盘式制动器优化设计（Optimal Design of a Disc Brake）

盘式制动器优化设计目标是最大限度减少制动器的质量和制动时间，共有四个设计参数分别是制动盘的内半径、外半径、啮合力和摩擦面数量，另外还有六个不等式约束，包括半径之间的最小距离、制动器的最大长度、压力、温度和扭矩限制。该问题的数学模型描述如下。

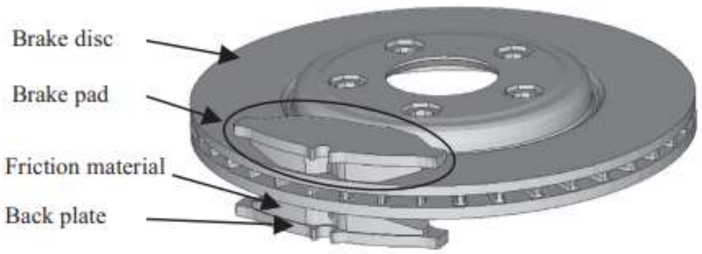

图34.28 盘式制动器示意图

$$Min.\quad \frac{4.9(x_2^2-x_1^2)(x_4-1)}{100000}$$

$$s.t.\begin{cases}\dfrac{980000(x_2^2-x_1^2)}{x_3x_4(x_2^3-x_1^3)}\le 32\\ 20-(x_2-x_1)\le 0\\ 2.5(x_4+1)-30\le 0\\ \dfrac{x_3}{\pi(x_2^2-x_1^2)}-0.4\le 0\\ \dfrac{0.00222x_3(x_2^3-x_1^3)}{(x_2^2-x_1^2)^2}-1\le 0\\ 900-\dfrac{0.0266x_3x_4(x_2^3-x_1^3)}{x_2^2-x_1^2}\le 0\end{cases}$$

其中：$55\le x_1\le 80, 75\le x_2\le 110, 1000\le x_3\le 3000, 2\le x_4\le 20,$。

代码：

```
Parameter x1=[55,80],x2=[75,110],x3=[1000,3000],x4=[2,20];
MinFunction 4.9*(x2^2-x1^2)*(x4-1)/100000;
        980000*(x2^2-x1^2)/(x3*x4*(x2^3-x1^3))<=32;
        20-(x2-x1)<=0;
        2.5*(x4+1)-30<=0;
        x3/(pi*(x2^2-x1^2))-0.4<=0;
        0.00222*x3*(x2^3-x1^3)/(x2^2-x1^2)^2-1<=0;
        900-0.0266*x3*x4*(x2^3-x1^3)/(x2^2-x1^2)<=0;
```

结果：

```
Objective Function (Min.): 0.1274
x1: 55
x2: 75
x3: 2180.97779620439
x4: 2

Constrained Functions:
1: 980000*(x2^2-x1^2)/(x3*x4*(x2^3-x1^3))-(32) = -29.7137312546048
2: 20-(x2-x1)-0 = 0
3: 2.5*(x4+1)-30-0 = -22.5
4: x3/(pi*(x2^2-x1^2))-0.4-0 = -0.132989694584926
5: 0.00222*x3*(x2^3-x1^3)/(x2^2-x1^2)^2-1-0 = -0.817001121925282
6: 900-0.0266*x3*x4*(x2^3-x1^3)/(x2^2-x1^2)-0 = -10501.9841510338
```

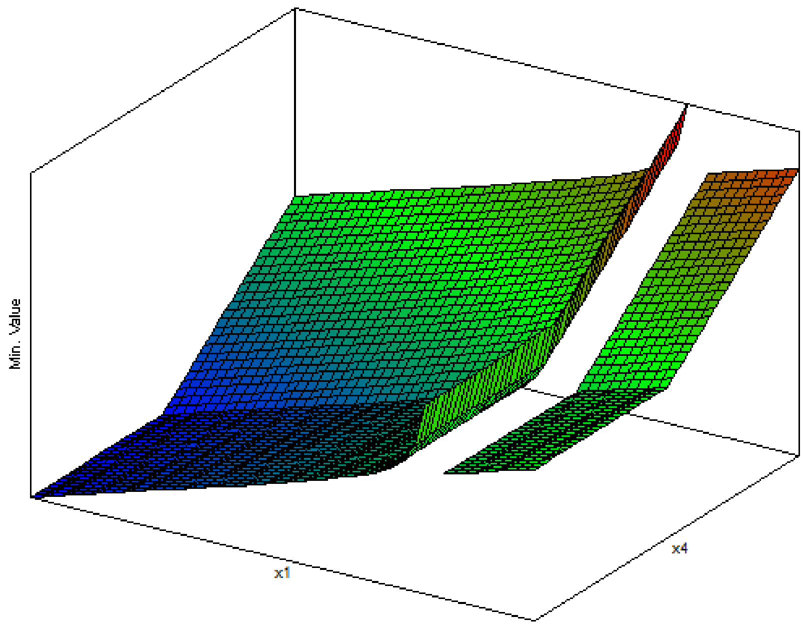

图34.29 盘式制动器优化设计问题目标函数随参数x1及x4变化三维图

34.3 小结

前述十五道经典工程优化问题，1stOpt都可以轻松高效求解，并均可获得稳定全局最优解。此外，1stOpt的求解程序代码几乎和模型描述的数学表达式一致，非常直观和易于理解，普通用户仅需花费很短时间即可掌握1stOpt的基本语法和用法，并可立即用于解决自己遇到的实际问题。