

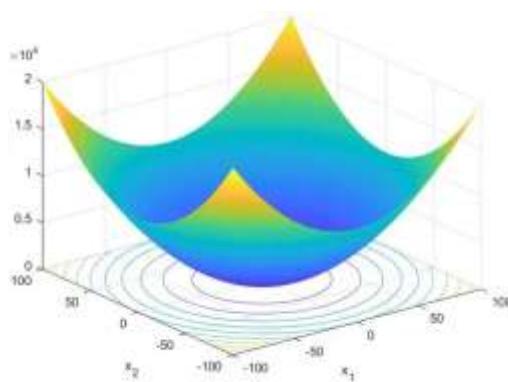
35. 六十道经典全局优化测试题集之 1stOpt 求解实现

35.1 引言

验证评价全局最优化算法及相关求解器最有效最权威的方式就是基于认可的测试题集（Benchmark）进行实战计算测试。目前市面上有很多全局优化相关的测试题集，这些测试题集一般都是多维且高度非线性，求解难度极高，同时由于其全局最优解均已给出、因此非常适用于全局最优算法的开发验证及对具有全局最优化求解能力的求解器进行计算效果评估。在此选取了六十道测试题，以 1stOpt 进行求解验证。

35.2 测试题集

35.2.1 Sphere Function

定义	$\text{Min. } \sum_{i=1}^n (x_i^2)$ <p>维度 30, 变量范围值 $[-100, 100]$, 全局最优值 0.</p>
三维示意图	
1stOpt 代码	Constant n=30; Parameter x(n)=[-100,100]; MinFunction Sum(i=1:n)(x[i]^2);
1stOpt 结果	Objective Function (Min.): 0 x1: 0 x2: 0 x3: 0 ...

35.2.2 Schwefel's Problem 2.22

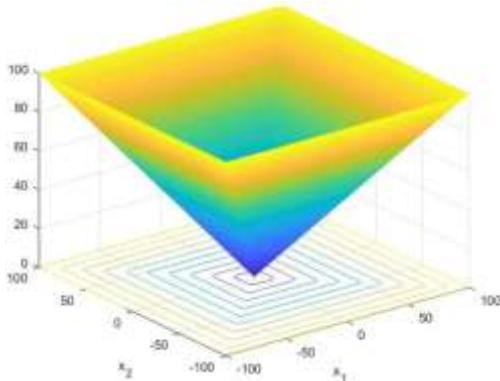
定义	$\text{Min. } \sum_{i=1}^n (x_i) + \prod_{i=1}^n (x_i)$ <p>维度 30, 变量范围值 [-10, 10], 全局最优值 0.</p>
三维示意图	
1stOpt 代码	<pre>Constant n=30; Parameter x(n)=[-10,10]; MinFunction Sum(i=1:n)(abs(x[i]))+Prod(i=1:n)(abs(x[i]));</pre>
1stOpt 结果	<p>Objective Function (Min.): 0</p> <p>x1: 0</p> <p>x2: 0</p> <p>x3: 0</p> <p>...</p>

35.2.3 Schwefel's Problem 1.2

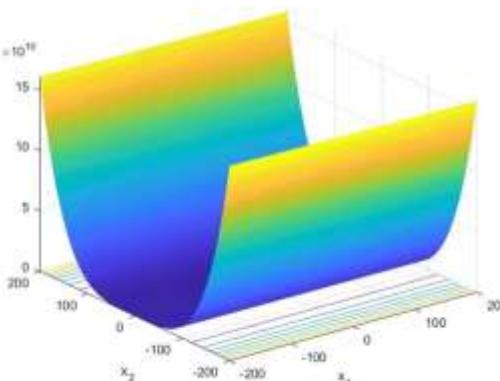
定义	$\text{Min. } \sum_{i=1}^n \left(\left(\sum_{j=1}^i x_j \right)^2 \right)$ <p>维度 30, 变量范围值 [-100, 100], 全局最优值 0.</p>
三维示意图	
1stOpt 代码	<pre>Constant n=30; Parameter x(n)=[-100,100]; MinFunction Sum(i=1:n)((Sum(j=1:i)(x[j]))^2);</pre>
1stOpt 结果	<p>Objective Function (Min.): 0</p> <p>x1: 0</p>

	x2: 0 x3: 0 ...
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35.2.4 Schwefel's Problem 2.21

定义	Min. $\max(x_i)$ $i = 1 \dots n$ 维度 30, 变量范围值 $[-100, 100]$, 全局最优值 0.
三维示意图	
1stOpt 代码	Constant n=30; Parameter x(n)=[-100,100]; MinFunction MaxIn(i=1:n)(abs(x[i]));
1stOpt 结果	Objective Function (Min.): 0 x1: 0 x2: 0 x3: 0 ...

35.2.5 Rosenbrock Function

定义	$\sum_{i=1}^{n-1} (100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2)$ 维度 30, 变量范围值 $[-30, 30]$, 全局最优值 0.
三维示意图	
1stOpt 代码	Constant n=30; Parameter x(n)=[-30,30]; MinFunction Sum(i=1:n-1)(100*(x[i+1]-x[i]^2)^2+(x[i]-1)^2);
1stOpt 结果	Objective Function (Min.): 1.34677481631058E-17 x1: 0.999999999950306

结果	x2: 0.999999999937923 x3: 0.999999999931753 ...
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352.6 Step Function

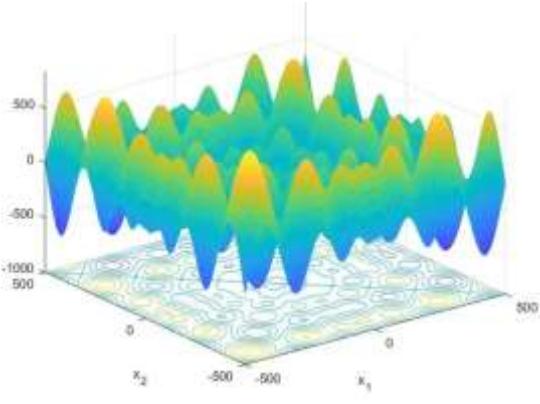
定义	$\sum_{i=1}^n ((x_i + 0.5)^2)$ <p>维度 30, 变量范围值 [-100, 100], 全局最优值 0.</p>
三维示意图	
1stOpt 代码	Constant n=30; Parameter x(n)=[-100,100]; MinFunction Sum(i=1:n)((x[i]+0.5)^2);
1stOpt 结果	Objective Function (Min.): 0 x1: -0.5 x2: -0.5 x3: -0.5 ...

352.7 Quartic Function

定义	$\sum_{i=1}^n (ix_i^4) + \text{random}$ <p>维度 30, 变量范围值 [-1.28, 1.28], 全局最优值 0.</p>
三维示意图	
1stOpt 代码	Constant n=30; Parameter x(n)=[-1.28,1.28], rnd=[0,1];

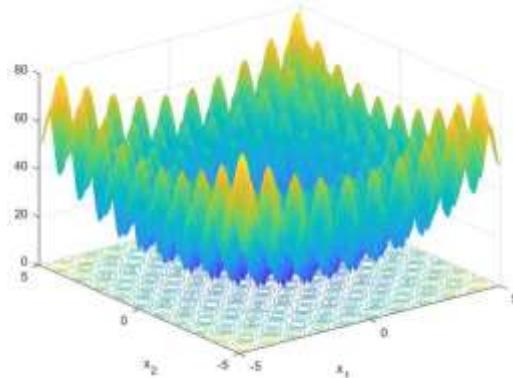
	MinFunction Sum(i=1:n)(i*x[i]^4)+rnd;
1stOpt 结果	Objective Function (Min.): 0 x1: 0 x2: 0 x3: 0 ...

35.2.8 Schwefel

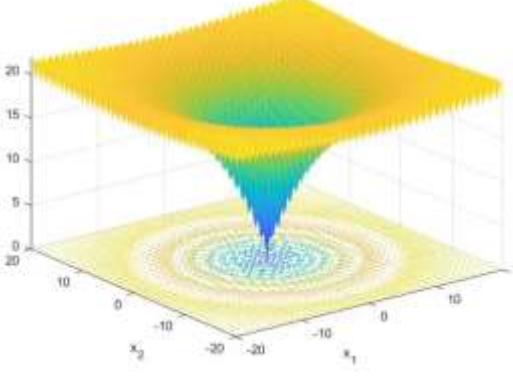
定义	$\sum_{i=1}^{30} (-x_i \sin(\sqrt{ x_i }))$ 维度 30, 变量范围值[-500,500], 全局最优值-418.9829*n.
三维示意图	
1stOpt 代码	Algorithm = DE1; Constant n=30; Parameter x(n)=[-500,500]; PassParameter obj=-418.982887272433*n; MinFunction Sum(i=1:n,x)(-x*sin(sqrt(abs(x))));
1stOpt 结果	Objective Function (Min.): -12569.486618173 x1: 420.968745599421 x2: 420.968746251568 x3: 420.968747159277 ...

35.2.9 Rastrigin Function

定义	$\sum_{i=1}^n (x_i^2 - 10\cos(2\pi x_i) + 10)$ 维度 30, 变量范围值[-5.12,5.12], 全局最优值 0.
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三维示意图	
1stOpt 代码	Constant n=30; Parameter x(n)=[-5.12,5.12]; MinFunction Sum(i=1:n,x)(x1^2-10*cos(2*pi*x)+10);
1stOpt 结果	Objective Function (Min.): 0 x1: 0 x2: 0 x3: 0 ...

35.2.10 Ackley Function

定义	$\text{Min. } -20\exp\left(-0.2\sqrt{\frac{\sum_{i=1}^n(x_i^2)}{n}}\right) - \exp\left(\frac{\sum_{i=1}^n(\cos(2\pi x_i))}{n}\right) + 20 + \exp(1)$ 维度 30, 变量范围值 [-32, 32], 全局最优值 0.
三维示意图	
1stOpt 代码	Constant n=30; Parameter x(n)=[-32,32]; MinFunction -20*exp(-0.2*sqrt(sum(i=1:n)(x[i]^2)/n))-exp(sum(i=1:n)(cos(2*pi*x[i])/n))+20+exp(1);
1stOpt 结果	Objective Function (Min.): 4.44089209850063E-16 x1: 0 x2: 0 x3: 0 ...

35.2.11 Griewank Function

定义	$\text{Min. } \frac{\sum_{i=1}^n (x_i^2)}{4000} - \prod_{i=1}^n \left(\cos\left(\frac{x_i}{\sqrt{i}}\right) \right) + 1$ <p>维度 30, 变量范围值 [-600, 600], 全局最优值 0.</p>
三维示意图	
1stOpt 代码	<pre>Constant n=30; Parameter x(n)=[-600,600]; MinFunction sum(i=1:n,x)(x^2)/4000-prod(i=1:n,x)(cos(x/sqrt(i)))+1;</pre>
1stOpt 结果	<p>Objective Function (Min.): 4.44089209850063E-16</p> <p>x1: 0 x2: 0 x3: 0 ...</p>

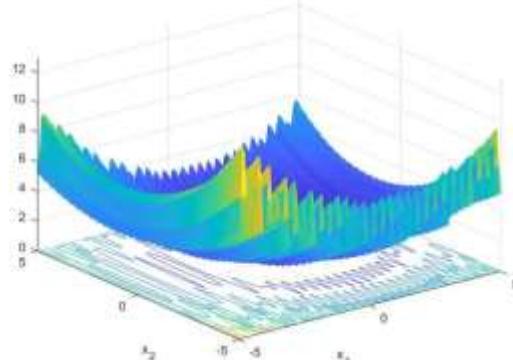
35.2.12 Penalized Function

定义	$\text{Min. } \frac{\pi}{n} \left(10 \left(\sin(\pi y(1)) \right)^2 + \sum_{i=1}^{n-1} ((y(i) - 1)^2) (1 + 10(\sin(\pi y(i+1)))^2) (y(30) - 1)^2 \right) + \sum_{i=1}^{30} (u(i))$ <p>其中:</p> $y(j) = 1 + \frac{x_j + 1}{4}$ $u(j) = \begin{cases} if(x_j > a, k(x_j - a)^m, if(x_j < -a, k(-x_j - a)^m, 0)) \end{cases}$ <p>维度 30, 变量范围值 [-50, 50], 全局最优值 0.</p>
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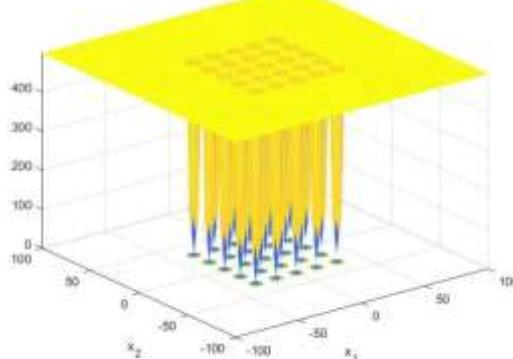
三维示意图	
1stOpt 代码	<pre>Algorithm = DE2; Constant n=30,a=10,k=100,m=4; ConstStr y(j)=1+(x[j]+1)/4, u(j)=if(x[j]>a,k*(x[j]-a)^m,if(x[j]<-a,k*(-x[j]-a)^m,0)); Parameter x(n)=[-50,50]; MinFunction (pi/n)*10*(sin(pi*y(1)))^2+sum(i=1:n-1)((y(i)-1)^2* (1+10*(sin(pi*y(i+1)))^2)+(y(n)-1)^2))+sum(i=1:n)(u(i));</pre>
1stOpt 结果	<p>Objective Function (Min.): 1.57044103551786E-32</p> <p>x1: -1 x2: -1 x3: -1</p>

352.13 Penalized 2 Function

定义	$\begin{aligned} \text{Min. } & 0.1 \left((\sin(3\pi x_1))^2 \right. \\ & + \sum_{i=1}^{n-1} \left((x_i - 1)^2 (1 + (\sin(3\pi x_{i+1}))^2) \right. \\ & \left. \left. + (x_n - 1)^2 (1 + (\sin(3\pi x_n))^2) \right) \right) + \sum_{i=1}^{30} (u(i)) \end{aligned}$ <p>其中:</p> $y(j) = 1 + \frac{x_j + 1}{4}$ $u(j) = if(x_j > a, k(x_j - a)^m, if(x_j < -a, k(-x_j - a)^m, 0))$ <p>维度 30, 变量范围值 [-50, 50], 全局最优值 0.</p>
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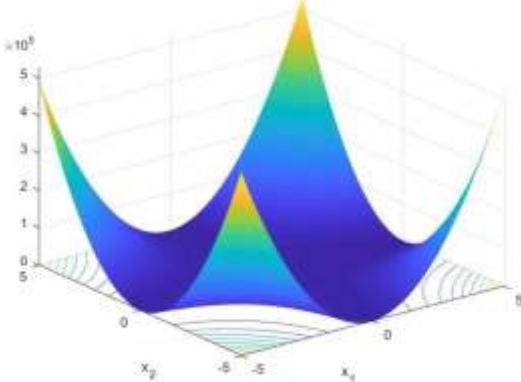
三维示意图	
1stOpt 代码	<pre> Algorithm = DE2; Constant n=30,a=5,k=100,m=4; ConstStr u(j)=if(x[j]>a,k*(x[j]-a)^m,if(x[j]<-a,k*(-x[j]-a)^m,0)); Parameter x(n)=[-50,50]; MinFunction 0.1*((sin(3*pi*x[1]))^2+sum(i=1:n)((x[i]-1)^2*(1+(sin(3*pi*x[i]))^2)+((x[n]-1)^2)*(1+(sin(2*pi*x[n]))^2))+sum(i=1:n)(u(i)); </pre>
1stOpt 结果	<p>Objective Function (Min.): 1.96599223184384E-32</p> <p>x1: 1 x2: 1 x3: 1 ...</p>

35.2.14 Foxholes Function

定义	$\text{Min.} \left(\frac{1}{500} + \sum_{j=1}^{25} \left(\frac{1}{j + \sum_{i=1}^2 ((x_i - a_{i,j})^6)} \right)^{-1} \right)$ <p>其中: $a(2,25)=[-32,-16,0,16,32,-32,-16,0,16,32,-32,-16,0,16,32,-32,-16,0,16,32,-32,-32,-32,-32,-32,-16,-16,-16,-16,-16,0,0,0,0,0,16,16,16,16,32,32,32,32,32];$ 维度 2, 变量范围值 [-65, 65], 全局最优值 0.998.</p>
三维示意图	
1stOpt 代码	<pre> Constant n=2; Constant a(2,25)=[-32,-16,0,16,32,-32,-16,0,16,32,-32,-16,0,16,32,-32,-16,0,16,32,-32,-32,-32,-32,-32,-16,-16,-16,-16,-16,0,0,0,0,0,16,16,16,16,32,32,32,32,32]; Parameter x(n)=[-65,65]; MinFunction (1/500+Sum(j=1:25)(1/(j+Sum(i=1:n)((x[i]-a[i,j])^6))))^-(-1); </pre>
1stOpt	Objective Function (Min.): 0.99800383779445

结果	x1: -31.9783337134144 x2: -31.9783336333451
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35.2.15 Kowalik Function

定义	$\text{Min. } \sum_{i=1}^{11} \left(a_i - \left(\frac{x_1(b_i^2 + x_2 b_i)}{b_i^2 + x_3 b_i + x_4} \right)^2 \right)$ <p>其中: $a=[0.1957,0.1947,0.1735,0.16,0.0844,0.0627,0.0456,0.0342,0.0323,0.0235,0.0246]$, $b^{-1}=[0.25,0.5,1,2,4,6,8,10,12,14,16]$; 维度 4, 变量范围值 [-5 , 5] , 全局最优值 0. 000307.</p>
三维示意图	
1stOpt 代码	<pre>Constant n=4; Constant a=[0.1957,0.1947,0.1735,0.16,0.0844,0.0627,0.0456,0.0342,0.0323,0.0235,0.0246], b=1/[0.25,0.5,1,2,4,6,8,10,12,14,16]; Parameter x(n)=[-5,5]; MinFunction sum(i=1:11,a,b)((a-((x1*(b^2+x2*b))/(b^2+x3*b+x4)))^2);</pre>
1stOpt 结果	<p>Objective Function (Min.): 0.000307485987805605 x1: 0.192833452958509 x2: 0.190836239164387 x3: 0.123117296287896 x4: 0.135765990157802</p>

35.2.16 Six-hump Camel-back Function

定义	$\text{Min. } 4x_1^2 - 2.1x_1^4 + \frac{x_1^6}{3} + x_1x_2 - 4x_2^2 + 4x_2^4$ <p>维度 2, 变量范围值 [-5 , 5] , 全局最优值-1. 3016.</p>
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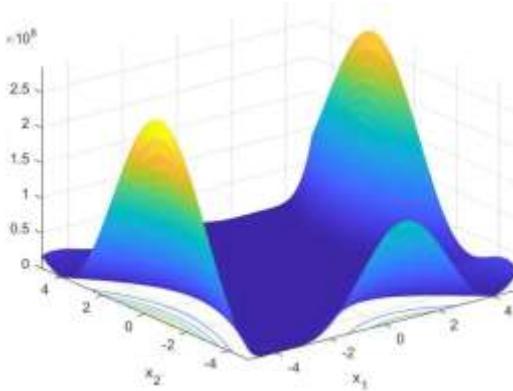
三维示意图	
1stOpt 代码	Constant n=2; Parameter x(n)=[-5,5]; MinFunction $4*x1^2-2.1*x1^4+x1^6+x1*x2-4*x2^2+4*x2^4$
1stOpt 结果	Objective Function (Min.): -1.03162845348988 x1: -0.0898420121058821 x2: 0.71265640283931

35.2.17 Branin Function

定义	$\text{Min. } \left(x_2 - \frac{x_1^2 5.1}{4\pi^2} + \frac{5}{\pi} x_1 - 6 \right)^2 + 10 \left(1 - \frac{1}{8\pi} \right) \cos(x_1) + 10$ 维度 2, 变量范围值 [-5 , 5] , 全局最优值 0. 398.
三维示意图	
1stOpt 代码	Constant n=2; Parameter x(n)=[-5,5]; MinFunction $(x_2-x_1^2*5.1/(4*(\pi^2))+5/\pi*x_1-6)^2+10*(1-1/(8*\pi))*\cos(x_1)+10$
1stOpt 结果	Objective Function (Min.): 0.397887357729738 x1: 3.1415926560214 x2: 2.27500000281768

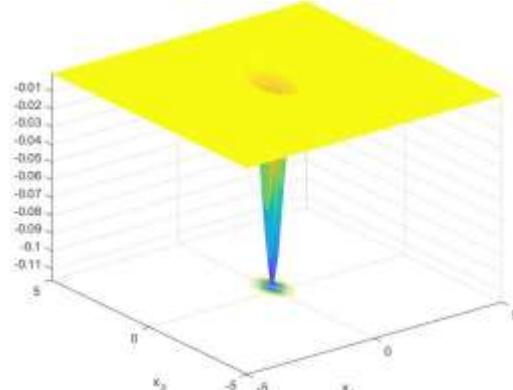
35.2.18 Goldstein-Price Function

定义	$\text{Min. } (1 + (x_1 + x_2 + 1)^2(19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1x_2 + 3x_2^2)(30 + (2x_1 - 3x_2)^2(18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1x_2 + 27x_2^2))$ 维度 2, 变量范围值 [-5 , 5] , 全局最优值 3.
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三维示意图	
1stOpt 代码	Constant n=2; Parameter x(n)=[-5,5]; MinFunction (1+(x1+x2+1)^2*(19-14*x1+3*(x1^2)-14*x2+6*x1*x2+3*x2^2))* (30+(2*x1-3*x2)^2*(18-32*x1+12*(x1^2)+48*x2-36*x1*x2+27*(x2^2)));
1stOpt 结果	Objective Function (Min.): 2.99999999999992 x1: 4.89938704426081E-10 x2: -0.999999999456401

35.2.19 Hartman 3 Function

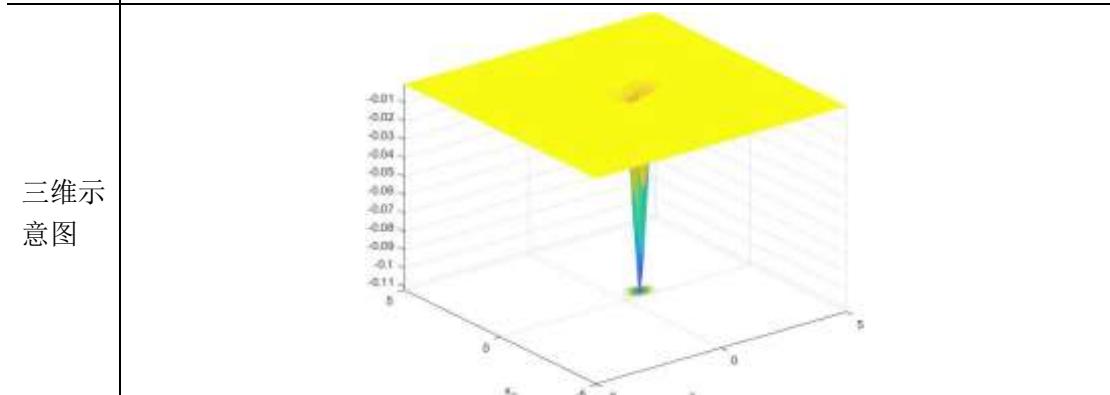
定义	$\text{Min.} - \sum_{i=1}^4 \left(c_i \exp \left(- \left(\sum_{j=1}^3 \left(a_{i,j} ((x_j - p_{i,j})^2) \right) \right) \right) \right)$ <p>其中:</p> $a(4, 3) = [3, 10, 30, 0.1, 10, 35, 3, 10, 30, 0.1, 10, 35],$ $c = [1, 1, 2, 3, 3, 2],$ $p(4, 3) = [.3689, .117, .2673, .4699, .4387, .747, .1091, .8732, .5547, .03815, .5743, .8828];$ <p>维度 2, 变量范围值 [1, 3], 全局最优值 -3.86.</p>
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三维示意图	
1stOpt	Constant n=2;

代码	<pre> Constant a(4,3)=[3,10,30, 0.1,10,35, 3,10,30, 0.1,10,35], c=[1,1.2,3,3.2], p(4,3)=[.3689,.117,.2673, .4699,.4387,.747, .1091,.8732,.5547, .03815,.5743,.8828]; Parameter x(n)=[-3,3]; MinFunction -sum(i=1:4)(c[i]*exp(-(sum(j=1:3)(a[i,j]*(x[j]-p[i,j])^2))));</pre>
1stOpt 结果	<p>Objective Function (Min.): -3.86278214782076</p> <p>x1: 0.1146143396929 x2: 0.555648849969821 x3: 0.852546953496436</p>

35.2.20 Hartman 6 Function

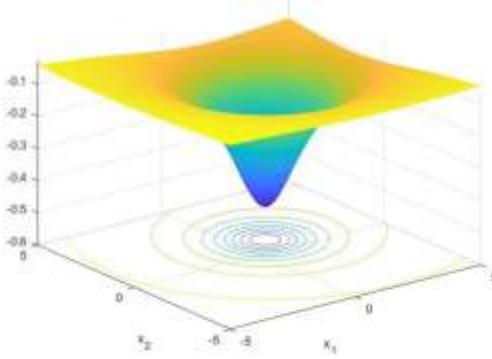
定义	$\text{Min.} - \sum_{i=1}^4 \left(c_i \exp \left(- \left(\sum_{j=1}^6 \left(a_{i,j} ((x_j - p_{i,j})^2) \right) \right) \right) \right)$ <p>其中:</p> <p>$a(4, 6) = [10, 3, 17, 3.5, 1.7, 8,$ $.05, 10, 17, .1, 8, 14,$ $3, 3.5, 1.7, 10, 17, 8,$ $17, 8, .05, 10, .1, 14],$ $c=[1, 1.2, 3, 3.2],$ $p(4, 6)=[.1312, .1696, .5569, .0124, .8283, .5886,$ $.2329, .4135, .8307, .3736, .1004, .9991,$ $.2348, .1415, .3522, .2883, .3047, .6650,$ $.4047, .8828, .8732, .5743, .1091, .0381];$ 维度 6, 变量范围值[0 ,1], 全局最优值-3. 32.</p>
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1stOpt 代码	<pre> Constant n=6; Constant a(4,6)=[10,3,17,3.5,1.7,8, .05,10,17,.1,8,14, 3,3.5,1.7,10,17,8, 17,8,.05,10,.1,14], c=[1,1.2,3,3.2],</pre>
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	<pre> p(4,6)=[.1312,.1696,.5569,.0124,.8283,.5886, .2329,.4135,.8307,.3736,.1004,.9991, .2348,.1415,.3522,.2883,.3047,.6650, .4047,.8828,.8732,.5743,.1091,.0381]; Parameter x(n)=[0,1]; MinFunction -sum(i=1:4)(c[i]*exp(-(sum(j=1:6)(a[i,j]^(x[j]-p[i,j])^2))));</pre>
1stOpt 结果	<p>Objective Function (Min.): -3.32199517158424</p> <p>x1: 0.201707618624006 x2: 0.146780946359304 x3: 0.476744852086078 x4: 0.275342390889558 x5: 0.311651875672778 x6: 0.657275162912526</p>

35.2.21 Shekel 5 Function

定义	$Min. = \sum_{i=1}^m \left(\left(\sum_{j=1}^n ((x_j - a_{j,i})^2 + c_i) \right)^{-1} \right)$ <p>其中:</p> $a(4, 10) = [4, 1, 8, 6, 3, 2, 5, 8, 6, 7,$ $4, 1, 8, 6, 7, 9, 3, 1, 2, 3.6,$ $4, 1, 8, 6, 3, 2, 5, 8, 6, 7,$ $4, 1, 8, 6, 7, 9, 3, 1, 2, 3.6],$ $c = [.1, .2, .2, .4, .4, .6, .3, .7, .5, .5];$ <p>维度 4, 变量范围值 [0, 10], 全局最优值 -10.1532.</p>
三维示意图	
1stOpt 代码	<pre> Constant n=4, m=5; Constant a(4,10)=[4,1,8,6,3,2,5,8,6,7, 4,1,8,6,7,9,3,1,2,3.6, 4,1,8,6,3,2,5,8,6,7, 4,1,8,6,7,9,3,1,2,3.6], c=[.1,.2,.2,.4,.4,.6,.3,.7,.5,.5]; Parameter x(n)=[0,10]; MinFunction -Sum(i=1:m)((Sum(j=1:4)((x[j]-a[j,i])^2)+c[i])^(-1));</pre>
1stOpt 结果	<p>Objective Function (Min.): -10.1531996790582</p> <p>x1: 4.0000371528205 x2: 4.00013327659262 x3: 4.00003715282014 x4: 4.0001332765927</p>

35.2.22 Shekel 7 Function

定义	$\text{Min.} = \sum_{i=1}^m \left(\left(\sum_{j=1}^n ((x_j - a_{j,i})^2 + c_i) \right)^{-1} \right)$ <p>其中:</p> $a(4, 10) = [4, 1, 8, 6, 3, 2, 5, 8, 6, 7, 4, 1, 8, 6, 7, 9, 3, 1, 2, 3.6, 4, 1, 8, 6, 3, 2, 5, 8, 6, 7, 4, 1, 8, 6, 7, 9, 3, 1, 2, 3.6],$ $c = [.1, .2, .2, .4, .4, .6, .3, .7, .5, .5];$ <p>维度 4, 变量范围值 [0, 10], 全局最优值 -10.4029.</p>
三维示意图	
1stOpt 代码	Constant n=4, m=7; Constant a(4,10)=[4,1,8,6,3,2,5,8,6,7, 4,1,8,6,7,9,3,1,2,3.6, 4,1,8,6,3,2,5,8,6,7, 4,1,8,6,7,9,3,1,2,3.6], c=[.1,.2,.2,.4,.4,.6,.3,.7,.5,.5]; Parameter x(n)=[0,10]; MinFunction =Sum(i=1:m)((Sum(j=1:4)((x[j]-a[j,i])^2)+c[i])^(-1));
1stOpt 结果	Objective Function (Min.): -10.4029153367777 x1: 4.00057281925618 x2: 3.99960620960477 x3: 4.00057281925589 x4: 3.9996062096047

35.2.23 Shekel 10 Function

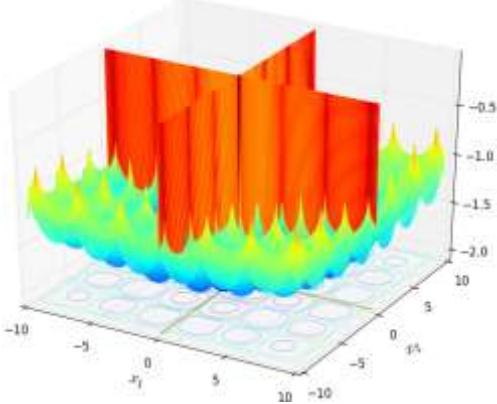
定义	$\text{Min.} = \sum_{i=1}^m \left(\left(\sum_{j=1}^n ((x_j - a_{j,i})^2 + c_i) \right)^{-1} \right)$ <p>其中:</p> $a(4, 10) = [4, 1, 8, 6, 3, 2, 5, 8, 6, 7, 4, 1, 8, 6, 7, 9, 3, 1, 2, 3.6, 4, 1, 8, 6, 3, 2, 5, 8, 6, 7, 4, 1, 8, 6, 7, 9, 3, 1, 2, 3.6],$
----	--

	<pre>c=[.1,.2,.2,.4,.4,.6,.3,.7,.5,.5]; 维度 4, 变量范围值[0 ,10], 全局最优值-10.5364.</pre>
三维示意图	
1stOpt 代码	<pre>Constant n=4, m=10; Constant a(4,10)=[4,1,8,6,3,2,5,8,6,7, 4,1,8,6,7,9,3,1,2,3,6, 4,1,8,6,3,2,5,8,6,7, 4,1,8,6,7,9,3,1,2,3,6]; c=[.1,.2,.2,.4,.4,.6,.3,.7,.5,.5]; Parameter x(n)=[0,10]; MinFunction =Sum(i=1:m)((Sum(j=1:4)((x[j]-a[j,i])^2)+c[i])^(-1));</pre>
1stOpt 结果	<p>Objective Function (Min.): -10.5364431534835 x1: 4.00074686821781 x2: 3.9995094801027 x3: 4.00074686831147 x4: 3.99950947999945</p>

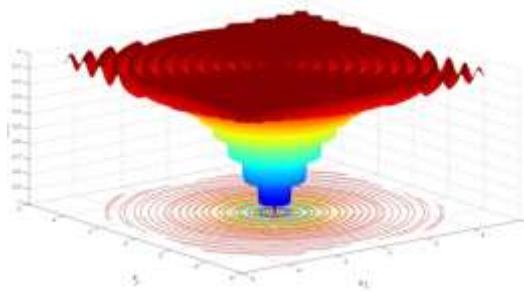
35.2.24 Bukin Function

定义	$\text{Min. } 100 \sqrt{ x_2 - 0.01x_1^2 } + 0.01 x_1 + 10 $ 维度为 2, $X1 \in [-15, -5]$, $X2 \in [-3, 3]$ 。该函数有许多局部极小值，它们都位于一个脊上。
三维示意图	
1stOpt 代码	<pre>Parameter x1=[-15,-5], x2=[-3,3]; MinFunction =100*sqrt(abs(x2-0.01*x1^2))+0.01*abs(x1+10);</pre>
1stOpt 结果	<p>Objective Function (Min.): 1.38893341272706E-13 x1: -10.00000000000139 x2: 1.00000000000278</p>

35.2.25 Cross-in-Tray Function

定义	$\text{Min. } -0.0001 \left(\left (\sin(x_1)\sin(x_2)) \left(\exp \left(\left 100 - \frac{\sqrt{x_1^2 + x_2^2}}{\pi} \right \right) \right) \right + 0.1 \right)^{0.1}$ <p>维度为 2, $X_i \in [-10, 10]$。</p>
三维示意图	
1stOpt 代码	<pre>Parameter x(2)=[-10,10]; ConstStr fact1 = sin(x1)*sin(x2), fact2 = exp(abs(100 - sqrt(x1^2+x2^2)/pi)); MinFunction -0.0001*(abs(fact1*fact2)+1)^0.1;</pre>
1stOpt 结果	<p>Objective Function (Min.): -2.06261187082274</p> <p>x1: 1.34940660378129 x2: -1.34940661850582</p>

35.2.26 Drop-Wave Function

定义	$\text{Min. } -\frac{1 + \cos(12\sqrt{x_1^2 + x_2^2})}{0.5(x_1^2 + x_2^2) + 2}$ <p>维度为 2, $X_i \in [-5.12, 5.12]$, 全局最优值-1。</p>
三维示意图	
1stOpt 代码	<pre>Parameter x(2)=[-5.12,5.12]; ConstStr frac1 = 1 + cos(12*sqrt(x1^2+x2^2)),frac2 = 0.5*(x1^2+x2^2)+2; MinFunction -frac1/frac2;</pre>
1stOpt 结果	<p>Objective Function (Min.): -1</p> <p>x1: 0 x2: 0</p>

35.2.27 Eggholder Function

定义	$\text{Min. } \left(\left(-(x_2 + 47) \sin \left(\sqrt{ x_2 + \frac{x_1}{2} + 47 } \right) \right) + \left(-x_1 \sin \left(\sqrt{ x_1 - (x_2 + 47) } \right) \right)$ <p>维度为 2, $X_i \in [-512, 512]$, 最优解为 -959.64</p>
三维示意图	
1stOpt 代码	<pre>Parameter x(2)=[-512,512]; ConstStr term1 = -(x2+47) * sin(sqrt(abs(x2+x1/2+47))), term2 = -x1 * sin(sqrt(abs(x1-(x2+47)))); MinFunction term1 + term2;</pre>
1stOpt 结果	<p>Objective Function (Min.): -959.640662720851 x1: 512 x2: 404.231804958642</p>

35.2.28 Gramacy & Lee (2012) Function

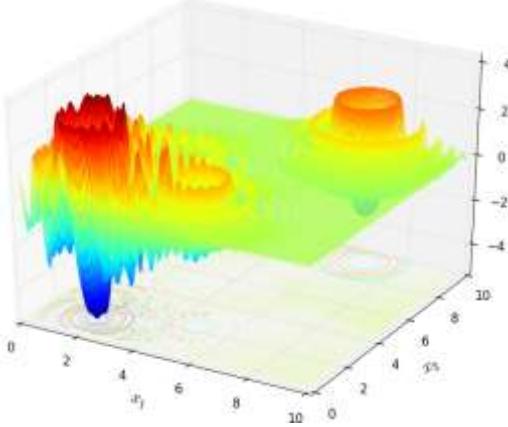
定义	$\text{Min. } \frac{\sin(10\pi x)}{2x} + (x - 1)^4$ <p>维度为 1, $X \in [0.5, 2.5]$。</p>
三维示意图	
1stOpt 代码	<pre>Parameter x=[0.5,2.5]; ConstStr term1 = sin(10*pi*x)/(2*x), term2 = (x-1)^4; MinFunction term1 + term2;</pre>
1stOpt 结果	<p>Objective Function (Min.): -0.8690111349895 x: 0.548563444572258</p>

35.2.29 Holder Table Function

定义	$-\left \sin(x_1)\cos(x_2)\exp\left(\left 1 - \frac{\sqrt{x_1^2 + x_2^2}}{\pi}\right \right)\right $ <p>四个全局极小值。维度为 2, $X_i \in [-10, 10]$。</p>
三维示意图	
1stOpt 代码	<pre>Parameter x(2)=[-10,10]; ConstStr fact1 = sin(x1)*cos(x2),fact2 = exp(abs(1 - sqrt(x1^2+x2^2)/pi)); MinFunction -abs(fact1*fact2);</pre>
1stOpt 结果	<p>Objective Function (Min.): -19.2085025678868</p> <p>x1: -8.05502346924964</p> <p>x2: -9.66459002411031</p>

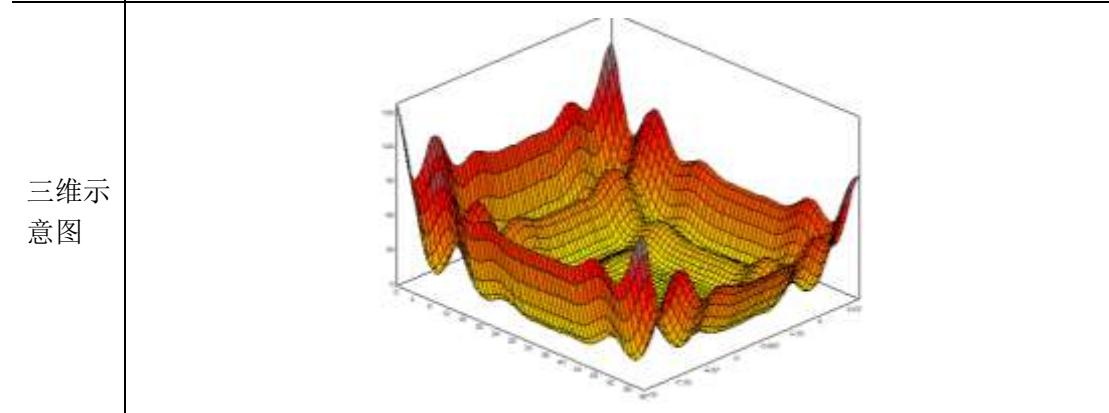
35.2.30 Langermann Function

定义	$\text{Min. } \sum_{i=1}^m \left(c_i \exp\left(-\frac{1}{\pi} \sum_{j=1}^n (x_j - a_{i,j})^2\right) \cos\left(\pi \sum_{j=1}^n (x_j - a_{i,j})^2\right) \right)$ <p>其中:</p> <p> $c = [1, 2, 5, 2, 3],$ $A(5, 2) = [3, 5,$ $5, 2,$ $2, 1,$ $1, 4,$ $7, 9];$ 维度 2, $X_i \in [0, 10]$。 </p>
----	--

三维示意图	
1stOpt 代码	<pre> Constant n=2, m=5; Constant c=[1,2,5,2,3], A(5,2)=[3,5, 5,2, 2,1, 1,4, 7,9]; Parameter x(n)=[0,10]; MinFunction Sum(i=1:m)(c[i]*exp(-1/pi*Sum(j=1:n)((x[j]-A[i,j])^2))*cos(pi*Sum(j=1:n)((x[j]-A[i,j])^2))); </pre>
1stOpt 结果	<p>Objective Function (Min.): -4.15580929184779 x1: 2.79340221022608 x2: 1.5972324991711</p>

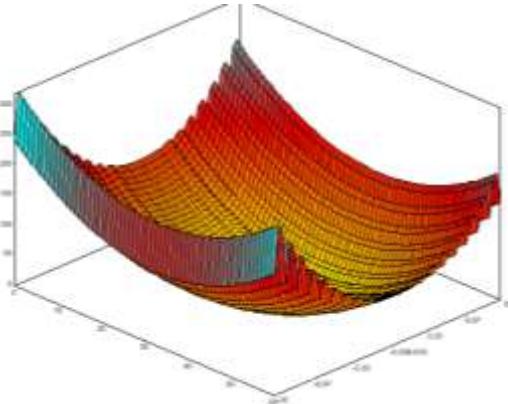
35.2.31 Levy Function

定义	$ \begin{aligned} & \text{Min. } \left(\sin(\pi w(1)) \right)^2 \\ & + \sum_{i=1}^{n-1} \left((w(i) - 1)^2 \left(1 + 10(\sin(\pi w(i) + 1))^2 \right) \right. \\ & \quad \left. + (w(n) - 1)^2 \left(1 + (\sin(2\pi w(n)))^2 \right) \right) \end{aligned} $ <p>其中: $w(j) = 1 + \frac{x_j - 1}{4}$</p> <p>维度 5, $x_i \in [-10, 10]$, 最优解 0。</p>
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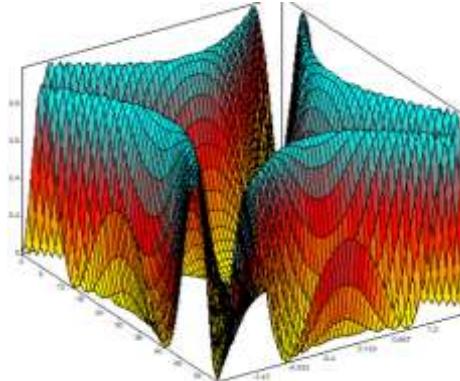
1stOpt 代码	Constant n=5; ConstStr w(j)=1+(x[j]-1)/4; Parameter x(n)=[-10,10]; MinFunction (sin(pi*w(1)))^2+Sum(i=1:n-1)((w(i)-1)^2*(1+10*(sin(pi*w(i)+1))^2))+(w(n)-1)^2*(1+(sin(2*pi*w(n)))^2);
1stOpt 结果	Objective Function (Min.): 5.86631508851742E-23 x1: 0.999999999992528 x2: 0.999999999983402 x3: 0.999999999983402 x4: 0.999999999983402 x5: 1

35.2.32 Levy Function N. 13

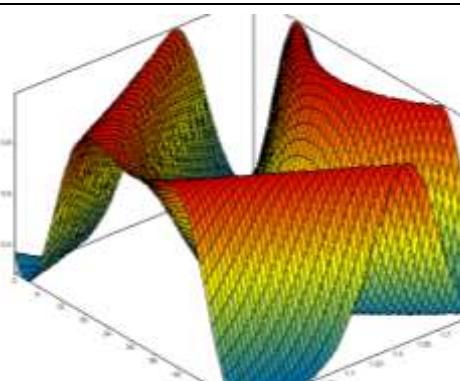
定义	$\text{Min. } (\sin(3\pi x_1))^2 + (x_1 - 1)^2(1 + (\sin(3\pi x_2))^2) + (x_2 - 1)^2(1 + (\sin(2\pi x_2))^2)$ 维度为 2, $x_i \in [-10, 10]$ 。
三维示意图	
1stOpt 代码	Parameter x(2)=[-10,10]; ConstStr term1 = (sin(3*pi*x1))^2, term2 = (x1-1)^2 * (1+(sin(3*pi*x2))^2), term3 = (x2-1)^2 * (1+(sin(2*pi*x2))^2); MinFunction term1 + term2 + term3;
1stOpt 结果	Objective Function (Min.): 1.34969464963992E-31 x1: 1 x2: 1

35.2.33 Schaffer Function N. 2 Function

定义	$\text{Min. } 0.5 + \frac{(\sin(x_1^2 - x_2^2))^2 - 0.5}{(1 + 0.001(x_1^2 + x_2^2))^2}$ 维度为 2, $x_i \in [-100, 100]$, 最优解为 0, $x=[0, 0]$ 。
----	--

三维示意图	
1stOpt 代码	<pre>Parameter x(2)=[-100,100]; ConstStr fact1 = (sin(x1^2-x2^2))^2 - 0.5, fact2 = (1 + 0.001*(x1^2+x2^2))^2; MinFunction 0.5 + fact1/fact2;</pre>
1stOpt 结果	<p>Objective Function (Min.): 0</p> <p>x1: 0 x2: 0</p>

35.2.34 Schaffer Function N. 4 Function

定义	$\text{Min. } 0.5 + \frac{\left(\cos(\sin(x_1^2 - x_2^2))\right)^2 - 0.5}{(1 + 0.001(x_1^2 + x_2^2))^2}$ <p>维度为 2, $X_i \in [-100, 100]$</p>
三维示意图	
1stOpt 代码	<pre>Parameter x(2)=[-100,100]; ConstStr fact1 = (cos(sin(abs(x1^2-x2^2))))^2 - 0.5, fact2 = (1 + 0.001*(x1^2+x2^2))^2; MinFunction 0.5 + fact1/fact2;</pre>
1stOpt 结果	<p>Objective Function (Min.): 0.29257863203598</p> <p>x1: -7.14280666514745E-8 x2: 1.25313183217751</p>

35.2.35 Shubert Function

定义	$\text{Min. } \left(\sum_{i=1}^5 (i \cos((i+1)x_1 + i)) \right) \left(\sum_{i=1}^5 (i \cos((i+1)x_2 + i)) \right)$
----	--

	维度为 2, $X_i \in [-10, 10]$, 全局最优解-186.7309
三维示意图	
1stOpt 代码	Parameter x(2)=[-10,10]; ConstStr term1=Sum(i=1:5)(i*cos((i+1)*x1+i)), term2=Sum(i=1:5)(i*cos((i+1)*x2+i)); MinFunction term1*term2;
1stOpt 结果	Objective Function (Min.): -186.730908831024 x1: 4.85805687883443 x2: -7.0835064079717

35.2.36 Bohachevsky Function

定义	$\text{Min. } x_1^2 + 2x_2^2 + (-0.3\cos(3\pi x_1)) + (-0.4\cos(4\pi x_2)) + 0.7$ 维度为 2, $X_i \in [-100, 100]$, 最优解为 0, $x=[0, 0]$
三维示意图	
1stOpt 代码	Parameter x(2)=[-100,100]; ConstStr fact1 = (cos(sin(abs(x1^2-x2^2))))^2 - 0.5, fact2 = (1 + 0.001*(x1^2+x2^2))^2; MinFunction 0.5 + fact1/fact2;
1stOpt 结果	Objective Function (Min.): 0.29257863203598 x1: -7.14280666514745E-8 x2: 1.25313183217751

35.2.37 Perm Function 0, d, β

定义	$\text{Min. } \sum_{i=1}^n \left(\left(\sum_{j=1}^n \left((j^i + 10) \left(x_j^i - \frac{1}{j^i} \right) \right)^2 \right) \right)$ 维度为 n, $X_i \in [-n, n]$
----	---

三维示意图	
1stOpt 代码	Constant n=5, b=10;//b=0.5; Parameter x(n)=[-n,n]; MinFunction Sum(i=1:n)((Sum(j=1:n)((j^i+b)*(x[j]^i-1/j^i)))^2);
1stOpt 结果	Objective Function (Min.): 1.33401739332335E-23 x1: 1.00000000000091 x2: 0.500000000011737 x3: 0.33333333270335 x4: 0.250000000105382 x5: 0.19999999946153

35.2.38 Rotated Hyper-Ellipsoid Function

定义	$\text{Min. } \sum_{i=1}^n \left(\sum_{j=1}^i (x_j^2) \right)$ 维度为 n, $X_i \in [-65.536, 65.536]$ 。
三维示意图	
1stOpt 代码	Constant n=10; Parameter x(n)=[-65.536, 65.536]; MinFunction Sum(i=1:n)(Sum(j=1:i)(x[j]^2));
1stOpt 结果	Objective Function (Min.): 0 x1: 0 x2: 0 x3: 0 ...

35.2.39 Sum of Different Powers Function

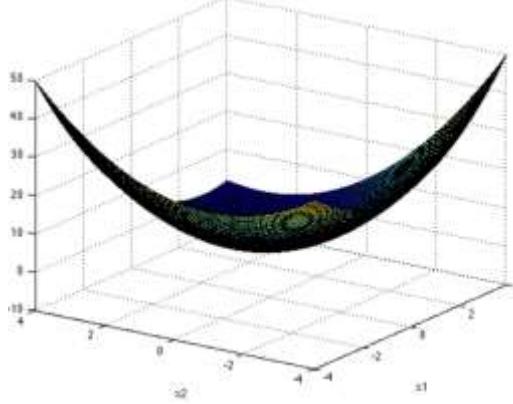
定义	$\text{Min. } \sum_{i=1}^n (x_i ^{i+1})$ 维度为 n, $x_i \in [-1, 1]$ 。
三维示意图	
1stOpt 代码	Constant n=10; Parameter x(n)=[-1,1]; MinFunction Sum(i=1:n)((abs(x[i])^(i+1)));
1stOpt 结果	Objective Function (Min.): 0 x1: 0 x2: 0 x3: 0 ...

35.2.40 Sum Squares Function

定义	$\text{Min. } \sum_{i=1}^n (ix_i^2)$ 维度为 n, $x_i \in [-10, 10]$
三维示意图	
1stOpt 代码	Constant n=10; Parameter x(n)=[-10,10]; MinFunction Sum(i=1:n)(i*x[i]^2);
1stOpt 结果	Objective Function (Min.): 0 x1: 0 x2: 0

	x3: 0 ...
--	--------------

35.2.41 Trid Function

定义	$\sum_{i=1}^n (x_i - 1)^2 - \sum_{i=2}^n (x_i x_{i-1})$ 维度为 n, $X_i \in [-n*n, n*n]$																				
三维示意图																					
1stOpt 代码	<pre>Constant n=10; Parameter x(n)=[-n*n,n*n]; PassParameter -n*(n+4)*(n-1)/6; //print out theoretically optimal values PassParameter For(i=1:n)(xx[i]=i*(n+1-i)); //print out Optimal parameter values MinFunction Sum(i=1:n)((x[i]-1)^2)-Sum(i=2:n)(x[i]*x[i-1]);</pre>																				
1stOpt 结果	<p>Objective Function (Min.): -210.00000000000003</p> <table> <tbody> <tr><td>x1:</td><td>10.0000002120105</td></tr> <tr><td>x2:</td><td>18.0000003503009</td></tr> <tr><td>x3:</td><td>24.0000003761177</td></tr> <tr><td>x4:</td><td>28.0000002491024</td></tr> <tr><td>x5:</td><td>30.0000004123977</td></tr> <tr><td>x6:</td><td>30.0000001069535</td></tr> <tr><td>x7:</td><td>28.000000363765</td></tr> <tr><td>x8:</td><td>24.0000000809627</td></tr> <tr><td>x9:</td><td>18.000000081472</td></tr> <tr><td>x10:</td><td>9.9999988192305</td></tr> </tbody> </table>	x1:	10.0000002120105	x2:	18.0000003503009	x3:	24.0000003761177	x4:	28.0000002491024	x5:	30.0000004123977	x6:	30.0000001069535	x7:	28.000000363765	x8:	24.0000000809627	x9:	18.000000081472	x10:	9.9999988192305
x1:	10.0000002120105																				
x2:	18.0000003503009																				
x3:	24.0000003761177																				
x4:	28.0000002491024																				
x5:	30.0000004123977																				
x6:	30.0000001069535																				
x7:	28.000000363765																				
x8:	24.0000000809627																				
x9:	18.000000081472																				
x10:	9.9999988192305																				

35.2.42 Booth Function

定义	$\text{Min. } (x_1 + 2x_2 - 7)^2 + (2x_1 + x_2 - 5)^2$ 维度为 2, $X_i \in [-10, 10]$
----	---

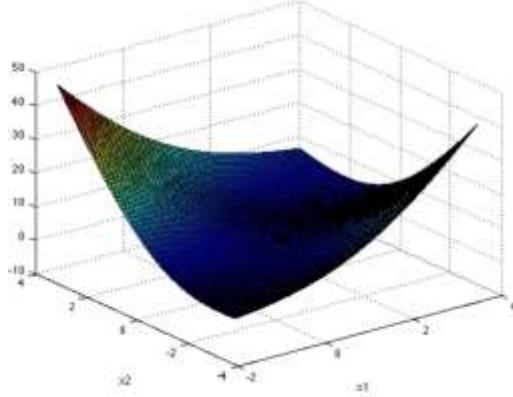
三维示意图	
1stOpt 代码	Constant n=2; Parameter x(n)=[-10,10]; MinFunction (x1+2*x2-7)^2+(2*x1+x2-5)^2;
1stOpt 结果	Objective Function (Min.): 0 x1: 1 x2: 3

35.2.43 Matyas Function

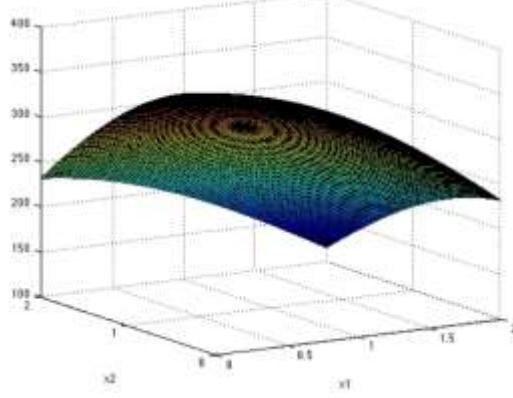
定义	$\text{Min. } 0.26(x_1^2 + x_2^2) - 0.48x_1x_2$ 维度为 2, $X_i \in [-10, 10]$
三维示意图	
1stOpt 代码	Constant n=2; Parameter x(n)=[-10,10]; MinFunction 0.26*(x1^2+x2^2)-0.48*x1*x2;
1stOpt 结果	Objective Function (Min.): 0 x1: 0 x2: 0

35.2.44 McCormick Function

定义	$\text{Min. } \sin(x_1 + x_2) + (x_1 - x_2)^2 - 1.5x_1 + 2.5x_2 + 1$ 维度为 2, $X1 \in [-1.5, 4]$, $X2 \in [-3, 4]$
----	--

三维示意图	
1stOpt 代码	Parameter x1=[-1.5,4], x2=[-3,4]; MinFunction sin(x1+x2)+(x1-x2)^2-1.5*x1+2.5*x2+1;
1stOpt 结果	Objective Function (Min.): -1.91322295498104 x1: -0.547197545365079 x2: -1.54719755013135

35.2.45 Power Sum Function

定义	$\text{Min. } \sum_{i=1}^4 \left(\left(\sum_{j=1}^4 x_j^i - b_i \right)^2 \right)$ <p>维度为 n=4, $X_i \in [0, d]$, $b=(8, 18, 44, 114)$。</p>
三维示意图	
1stOpt 代码	Constant n=4, b=[8,18,44,114] Parameter x(n)=[0,n]; MinFunction Sum(i=1:n)((Sum(j=1:n)(x[j]^i)-b[i])^2);
1stOpt 结果	Objective Function (Min.): 7.77501308185829E-27 x1: 2.99999999999973 x2: 1.9999991257255 x3: 2.00000087427418 x4: 1.000000000000065

35.2.46 Zakharov Function

定义	$\text{Min. } \sum_{i=1}^n x_i^2 + \left(\sum_{i=1}^n 0.5ix_i \right)^2 + \left(\sum_{i=1}^n 0.5ix_i \right)^4$
----	---

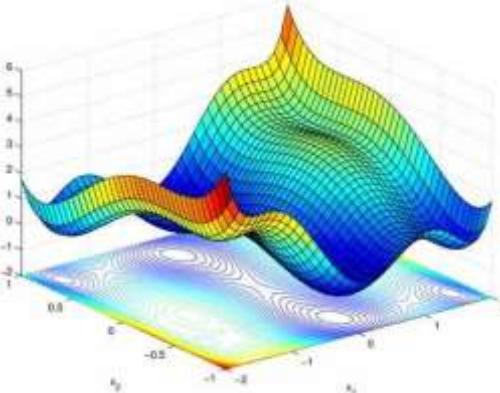
	维度为 n, $X_i \in [-5, 10]$ 。
三维示意图	
1stOpt 代码	<pre>Constant n=5; ConstStr f=Sum(i=1:n)(0.5*i*x[i]); Parameter x(n)=[-5,10]; MinFunction Sum(i=1:n)(x[i]^2)+f^2+f^4;</pre>
1stOpt 结果	<p>Objective Function (Min.): 0</p> <p>x1: 0 x2: 0 x3: 0 x4: 0 x5: 0</p>

35.2.47 Three-Hump Camel Function

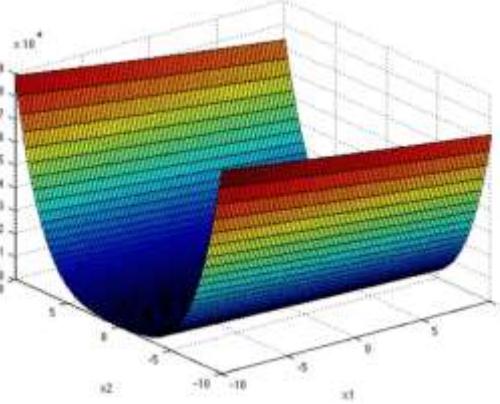
定义	$2x_1^2 - 1.05x_1^4 + \frac{x_1^6}{6} + x_1x_2 + x_2^2$ <p>维度为 2, $X_i \in [-5, 5]$</p>
三维示意图	
1stOpt 代码	<pre>Constant n=2; Parameter x(n)=[-5,5]; MinFunction 2*x1^2-1.05*x1^4+x1^6/6+x1*x2+x2^2;</pre>
1stOpt 结果	<p>Objective Function (Min.): 0</p> <p>x1: 0 x2: 0</p>

35.2.48 Six-Hump Camel Function

定义	$\text{Min. } \left(4 - 2.1x_1^2 + \frac{x_1^4}{3}\right)x_1^2 + x_1x_2 + (-4 + 4x_2^2)x_2^2$
----	---

	维度为 2, $X_i \in [-3, 3]$
三维示意图	
1stOpt 代码	Parameter x1=[-3,3], x2=[-2,2]; MinFunction (4-2.1*x1^2+(x1^4)/3)*x1^2+x1*x2+(-4+4*x2^2)*x2^2;
1stOpt 结果	Objective Function (Min.): -1.03162845348988 x1: 0.0898420131596889 x2: -0.712656402697174

35.2.49 Dixon-Price Function

定义	$\text{Min. } (x_1 - 1)^2 + \sum_{i=2}^{10} i(2x_i^2 - x_{i-1})^2$ 维度为 n, $X_i \in [-10, 10]$
三维示意图	
1stOpt 代码	Constant n=10; Parameter x(n)=[-10,10]; PassParameter For(i=1:n)(xx[i]=2^(-(2^i-2)/2^i)); MinFunction (x1-1)^2+Sum(i=2:n)(i*(2*x[i]^2-x[i-1])^2);
1stOpt 结果	Objective Function (Min.): 1.16818915484944E-20 x1: 0.999999999868227 x2: 0.707106781116665 x3: 0.594603557449522 x4: 0.5452538662869 x5: 0.522136891170456 x6: 0.510948574285353 x7: 0.505444642985836 x8: 0.502714950518905 x9: 0.501355637491785 x10: 0.500677359920016

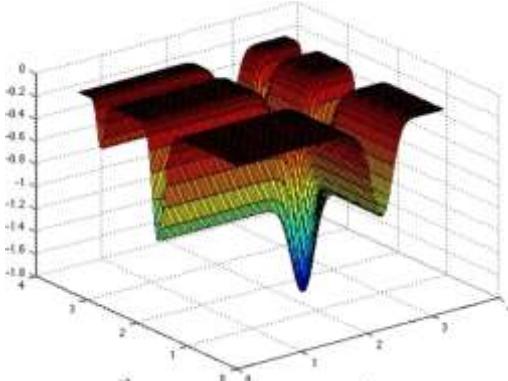
35.2.50 De Jong Function N. 5

定义	$\text{Min.} \left(0.002 + \sum_{i=1}^{25} \left(\frac{1}{i + (x_1 - a_{1,i})^6 + (x_2 - a_{2,i})^6} \right) \right)^{-1}$ <p>维度为 2, $X_i \in [-65.536, 65.536]$。</p>
三维示意图	
1stOpt 代码	<pre>Constant n=2; Constant a(2,25)=[-32,-16,0,16,32,-32,-16,0,16,32,-32,-16,0,16,32,-32,-16,0,16,32,-32,-16,0,16,32,-32,-32,-32,-32,-32,-32,-16,-16,-16,-16,0,0,0,0,16,16,16,16,32,32,32,32,32]; Parameter x(n)=[-65.536,65.536]; MinFunction (0.002+Sum(i=1:25)(1/(i+(x1-a[1,i])^6+(x2-a[2,i])^6)))^(-1);</pre>
1stOpt 结果	<p>Objective Function (Min.): 0.99800383779445 x1: -31.9783353453833 x2: -31.9783323160425</p>

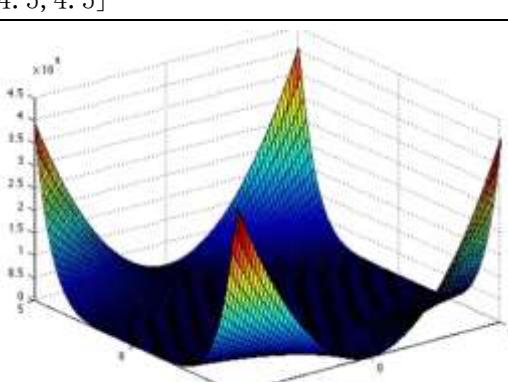
35.2.51 Easom Function

定义	$\text{Min.} - \cos(x_1)\cos(x_2)\exp(-(x_1 - \pi)^2 - (x_2 - \pi)^2)$ <p>维度为 2, $X_i \in [-100, 100]$</p>
三维示意图	
1stOpt 代码	<pre>Constant n=2; Parameter x(n)=[-100,100]; MinFunction -cos(x1)*cos(x2)*exp(-(x1-pi)^2-(x2-pi)^2);</pre>
1stOpt 结果	<p>Objective Function (Min.): -1 x1: 3.14159265146846 x2: 3.14159265146846</p>

35.2.52 Michalewicz Function

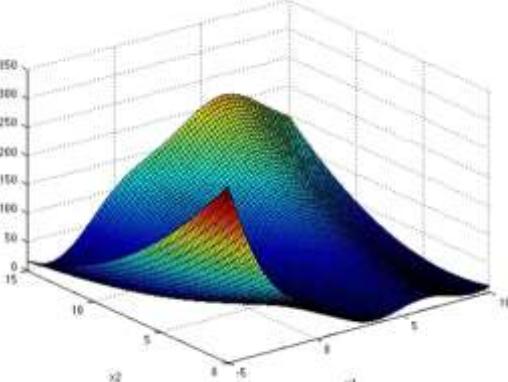
定义	$\text{Min. } - \sum_{i=1}^n \sin(x_i) \left(\sin\left(\frac{ix_i^2}{\pi}\right) \right)^{2m}$ <p>维度为 n, $X_i \in [0, \pi]$</p>
三维示意图	
1stOpt 代码	<pre>Algorithm = RGO2; Constant n=10,m=10; Parameter x(n)=[0,pi]; MinFunction =Sum(i=1:n)(sin(x[i])*(sin(i*x[i]^2/pi))^(2*m));</pre>
1stOpt 结果	<p>Objective Function (Min.): -9.66015171564135</p> <p>x1: 2.2029055183045 x2: 1.57079632987288 x3: 1.28499157131628 x4: 1.9230584696599 x5: 1.7204697725674 x6: 1.57079632668502 x7: 1.45441397169618 x8: 1.75608652165324 x9: 1.65571741677492 x10: 1.57079632652128</p>

35.2.53 Beale Function

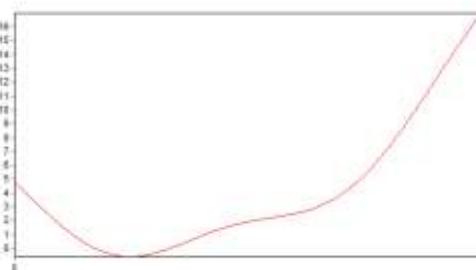
定义	$\text{Min. } (1.5 - x_1 + x_1 x_2)^2 + (2.25 - x_1 + x_1 x_2^2)^2 + (2.625 - x_1 + x_1 x_2^3)^2$ <p>维度为 2, $X_i \in [-4.5, 4.5]$</p>
三维示意图	
1stOpt	Parameter x(2)=[-4.5,4.5];

代码	<code>MinFunction (1.5 - x1 + x1*x2)^2+(2.25 - x1 + x1*x2^2)^2+(2.625 - x1 + x1*x2^3)^2;</code>
1stOpt 结果	Objective Function (Min.): 1.7779593600904E-26 x1: 3.00000000000033 x2: 0.500000000000083

35.2.54 Branin Function

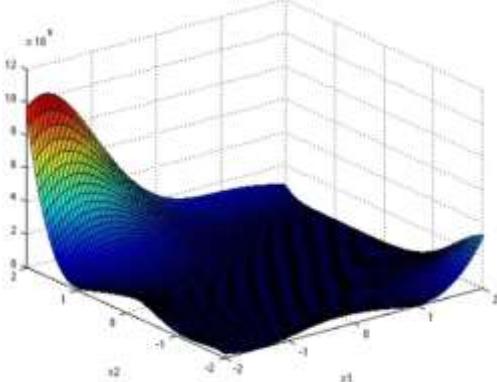
定义	$\text{Min. } a(x_2 - bx_2^2 + cx_1 - r)^2 + s(1 - t)\cos(x_1) + s$ 其中: $a=1$ 、 $b=5.1/(4\pi^2)$ 、 $c=5/\pi$ 、 $r=6$ 、 $s=10$ 和 $t=1/(8\pi)$ 。 维度为 2, $X1 \in [-5, 10]$, $X2 \in [0, 15]$ 。
三维示意图	
1stOpt 代码	Constant $a=1, b=5.1/(4*\pi^2), c=5/\pi, r=6, s=10, t=1/(8*\pi)$; Parameter $x1=[-5, 10], x2=[0, 15]$; <code>MinFunction a*(x2-b*x2^2+c*x1-r)^2+s*(1-t)*cos(x1)+s;</code>
1stOpt 结果	Objective Function (Min.): 0.397887357729738 x1: 3.14159266123008 x2: 6.56104390014507

35.2.55 Forrester et al. (2008) Function

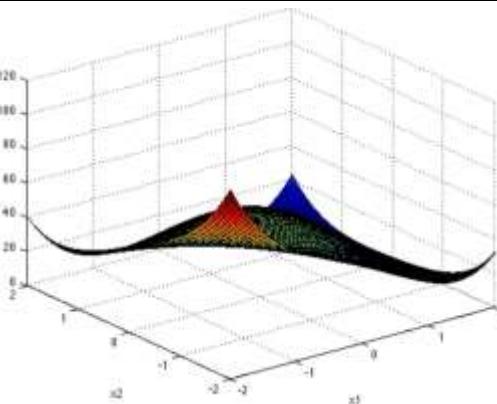
定义	$\text{Min. } (6x - 2)^2 + \sin(12x - 4)$ 维度为 1, $Xi \in [0, 1]$
三维示意图	
1stOpt 代码	Parameter $x=[0,1]$; <code>MinFunction (6*x-2)^2+sin(12*x-4);</code>
1stOpt 结果	Objective Function (Min.): -0.592074001277944 x: 0.247511122600896

35.2.56 Goldstein-Price Function

定义	$\text{Min. } (1 + s_1s_2)(30 + s_3s_4)$
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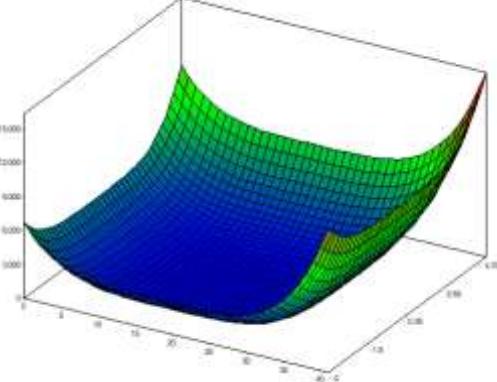
	<p>其中: $s_1 = (x_1 + x_2 + 1)^2$, $s_2 = 19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1x_2 + 3x_2^2$ $s_3 = (2x_1 - 3x_2)^2$, $s_4 = 18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1x_2 + 27x_2^2$ 维度为 2, $X_i \in [-2, 2]$</p>
三维示意图	
1stOpt 代码	<pre>Constant n=2; ConstStr s1=(x1+x2+1)^2, s2=19-14*x1+3*x1^2-14*x2+6*x1*x2+3*x2^2, f1=1+s1*s2, s3=(2*x1-3*x2)^2, s4=18-32*x1+12*x1^2+48*x2-36*x1*x2+27*x2^2, f2=30+s3*s4; Parameter x(n)=[-2,2]; MinFunction f1*f2;</pre>
1stOpt 结果	<p>Objective Function (Min.): 2.99999999999992 x1: -4.39855001928663E-9 x2: -1.000000000000659</p>

35.2.57 Perm Function d, β

定义	$\text{Min. } \sum_{i=1}^n \left(\sum_{j=1}^n (j^i + \beta) \left(\left(\frac{x_j}{j} \right)^i - 1 \right) \right)^2$
三维示意图	
1stOpt 代码	<pre>Constant n=5, b=0.5; Parameter x(n)=[-n,n]; MinFunction Sum(i=1:n)((Sum(j=1:n)((j^i+b)*((x[j]/j)^i-1)))^2);</pre>
1stOpt 结果	<p>Objective Function (Min.): 1.11477914252995E-20 x1: 0.999999999854174</p>

结果	x2: 2.00000000044031 x3: 2.99999999948293 x4: 4.00000000028216 x5: 4.9999999994043
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35.2.58 Powell Function

定义	$\text{Min. } \sum_{i=1}^{n/4} (s_1 + s_2 + s_3 + s_4)$ <p>其中: $s_1 = (x_{4i-3} + 10x_{4i-2})^2, s_2 = 5(x_{4i-1} - x_{4i})^2,$ $s_3 = (x_{4i-2} - 2x_{4i-1})^4, s_4 = 10(x_{4i-3} - x_{4i})^4$ 维度为 n, $X_i \in [-4, 5]$</p>
三维示意图	
1stOpt 代码	<pre>Constant n=12; Parameter x(n)=[-4,5]; ConstStr s1 = (x[4*i-3] + 10*x[4*i-2])^2, s2 = 5 * (x[4*i-1] - x[4*i])^2, s3 = (x[4*i-2] - 2*x[4*i-1])^4, s4 = 10 * (x[4*i-3] - x[4*i])^4, s = s1 + s2 + s3 + s4; MinFunction Sum(i=1:n/4)(s);</pre>
1stOpt 结果	<p>Objective Function (Min.): 0</p> <p>x1: 0 x2: 0 x3: 0 ...</p>

35.2.59 Styblinski-Tang Function

定义	$\text{Min. } \frac{1}{2} \sum_{i=1}^n (x_i^4 - 16x_i^2 + 5x_i)$ <p>维度为 n, $X_i \in [-5, 5]$</p>
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三维示意图	
1stOpt 代码	<pre>Constant n=15; Parameter x(n)=[-5,5]; PassParameter obj=-39.1661657*n; MinFunction 1/2*Sum(i=1:n,x)(x^4-16*x^2+5*x);</pre>
1stOpt 结果	<p>Objective Function (Min.): -587.492485556571 x1: -2.90353402266191 x2: -2.90353402266191 x3: -2.90353402266191 ...</p>

35.2.60 Sine Wave Function

定义	$\text{Min. } \sum_{i=1}^{n-1} \left(\frac{\left(\sqrt{x_{i+1}^2 + x_i^2} - 0.5 \right)^2}{(0.001(x_{i+1}^2 + x_i^2) + 1)^2} + 0.5 \right)$ <p>维度为 n, $X_i \in [-100, 100]$, 最优解为: $-0.5+0.5*n$</p>
三维示意图	
1stOpt 代码	<pre>Constant n=25; Parameters x(1:n)=[-100,100]; MinFunction Sum(i=1:n-1)(sqr(sqrt(x[i+1]^2+x[i]^2)-0.5)/sqr(0.001*(x[i+1]^2+x[i]^2)+1)+0.5);</pre>
1stOpt 结果	<p>Objective Function (Min.): 12 x1: 0.353540125970658 x2: 0.353566670562887 x3: 0.35354011103622 ...</p>

35.3 小结

基于 1stOpt, 对六十道经典全局优化测试题进行了计算评估, 从实现代码及计算结果看, 1stOpt 不仅能以简单直观的代码实现所有的问题, 而且均能轻松获得认证的全局最优解, 这些测试题集对 1stOpt 而言, 几乎不存在难度, 或者说已失去了“挑战”的意义。